

Vibration-Rotation Interaction in Engineering Problems. I. Normal Mode of Vibration and Coriolis Coupling Coefficients

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Abstract

A theoretical work of vibration-rotation interaction of a basic mechanical problem is presented here. The normal coordinate method is adopted to analyze the vibration of a thin metal plate using a simple model of a planar D_{3h} symmetry. Four types of normal modes of vibration which are totally symmetric (A'_1), perpendicular to the symmetry plane (A''_2), and doubly degenerated asymmetric (E'_{3a}, E'_{4a} ; E'_{3b}, E'_{4b}) vibrations are discussed.

The matrix elements of Coriolis coupling coefficients are also worked out on the interaction between vibration and rotation at the same model of symmetry species in this paper.

1. Introduction

In the last decade there has been a remarkable progress in the understanding of the nature of vibration problems of mechanical rotators. Some of laborious works^(1,2) are related to the vibration-rotation interactions of a running ball bearing and others^(3,4,5,6,7,8) are concerned with vibrations of the asymmetric rotators under the various conditions. They are all supposed to fulfill the requirements of the theoretical and practical engineering applications.

The solid state associated with a definite crystalline form is considered to be a completely regular arrangement of the atoms or molecules of which the substance is constituted. It is reasonable to regard that the solid body is composed of a perfectly elastic matter within the limit of the fundamental law known as Hooke's law. Now, it may be a worthwhile attempt to introduce the general theory of dynamics⁽⁹⁾ into the deformations of a body. A procedure of the normal coordinate method may be one of the most powerful tool to analyze the vibration problems.

In spite of the lack of sufficient informations for analyzing the dynamical treatment of the elastic solid body, the recent developments of the theory of elasticity^(10,11) and the rheological study of metals⁽¹²⁾ will give us an encouragement to challenge for the problems of vibration analysis.

One of the purposes of the present work is to discuss an applicability of the general theory of dynamics to the deformations or vibrations of a solid body and to elucidate the characteristic features of the interactions between vibration and rotation about an elastic body under the action of forces.

2. Descriptions of the System

In order to discuss the vibration and the interactions between vibration and rotation of a system, an attention may be focused on the fundamental theory of dynamics which is familiar and straightforward mathematics. For analyzing the complex deformations and movements of engineering materials, it is a common way to resolve the applied forces into components which are convenient to evaluate. This will be adequately accomplished by introducing the knowledges of the normal coordinate analysis. The normal coordinate method developed recently⁽¹³⁾ is considered to be more powerful technique than the analysis using the cartesian coordinates.

The method described here is somewhat skillful to make use of vectorial notation and matrix algebra by an aid of the fundamental group theory⁽¹⁴⁾.

Unfortunately, a practice becomes cumbersome because the separation of an equation of motion into vibrational and rotational movements is essentially difficult. In cutting works, as a simple example, a thin metal plate clamped tightly by the chuck of a lathe generates sometimes noisy sound at a choice of wrong cutting speed. It may be given a fairly smooth cutting surface without any noise, provided no vibrational movement is introduced during the lathe work.

The deformations of a body under the applied forces are described mathematically in the changes of a position denoted by α of any point and by \mathbf{r}_α of its radius vector in some coordinate system. The displacement vector ρ_α due to the deformation is given as

$$\rho_\alpha = \mathbf{r}_\alpha - \mathbf{a}_\alpha \quad (1)$$

in which \mathbf{a}_α is the equilibrium position vector of α -th point. The center of mass of the body is located from the origin by the vector \mathbf{R} in the space-fixed axis system. If the rotating axes have the angular velocity of $\boldsymbol{\omega}$ and the vector \mathbf{v}_α , which has the components \dot{x}_α , \dot{y}_α , and \dot{z}_α in cartesian system, the velocity of the α -th point in space is written in the form

$$\mathbf{V} = \dot{\mathbf{R}} + (\boldsymbol{\omega} \times \mathbf{r}_\alpha) + \mathbf{v}_\alpha \quad (2)$$

The kinetic energy of the whole system is given by

$$\begin{aligned} 2T = \sum_{\alpha} m \mathbf{V}^2 = & \dot{\mathbf{R}}^2 \sum_{\alpha} m_{\alpha} + \sum_{\alpha} m_{\alpha} (\boldsymbol{\omega} \times \mathbf{r}_{\alpha}) \cdot (\boldsymbol{\omega} \times \mathbf{r}_{\alpha}) \\ & + \sum_{\alpha} m_{\alpha} \mathbf{v}_{\alpha}^2 + 2 \dot{\mathbf{R}} \cdot \boldsymbol{\omega} \times \sum_{\alpha} m_{\alpha} \mathbf{r}_{\alpha} + 2 \dot{\mathbf{R}} \cdot \sum_{\alpha} m_{\alpha} \mathbf{v}_{\alpha} + 2 \boldsymbol{\omega} \cdot \sum_{\alpha} (m_{\alpha} \mathbf{r}_{\alpha} \times \mathbf{v}_{\alpha}) \end{aligned} \quad (3)$$

in which the first term is the translation kinetic energy, whereas the second and third terms are the purely rotational and purely vibrational kinetic energies, respectively. The fourth and fifth terms are the interactions of the translation with rotation and with vibration. The last term is concerned with the kinetic energy of the interactions between rotation and vibration, the so-called Coriolis energy.⁽¹⁵⁾ This term is of much importance in the study of mechanical vibration which merits of further attention. All terms including $\dot{\mathbf{R}}$ can be neglected because of the existence of the conditions of Eckart's⁽¹⁶⁾ and of the movements around the center of gravity⁽¹⁷⁾.

To describe the dynamical system, a simple model with a symmetry point group of D_{3d}

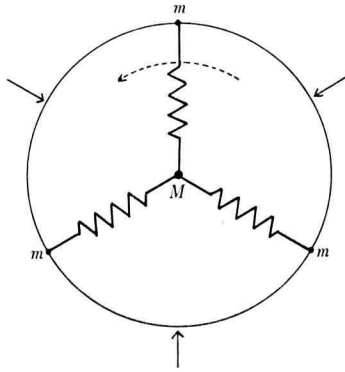


Fig. 1 Schematic diagram of the system.

will be used. In a plane system, three of point masses which have the same hypothetical weight m are connected with a point mass M at the center of the plane by three of weightless springs, as shown in Fig. 1.

This plane system is fastened by three forces facing each other, e.g., the chuck of a lathe as represented by fine arrows in Fig. 1. It is turned round about the center of plane shown a circular dotted arrow.

At first, it might be discussed that what kind of movements would be generated in the system mentioned above when a force were applied at any point of the spring.

3. Normal Mode of Vibration

As a consequence of the general theory of dynamics, the kinetic energy is given by

$$2T = \sum_{\alpha} m_{\alpha} \left[\left(\frac{d\Delta x_{\alpha}}{dt} \right)^2 + \left(\frac{d\Delta y_{\alpha}}{dt} \right)^2 + \left(\frac{d\Delta z_{\alpha}}{dt} \right)^2 \right] \quad (4)$$

In the normal coordinate language, the cartesian displacement coordinates \mathbf{X} (i.e., $\Delta x_1, \dots, \Delta z_{3\alpha}$) are necessarily replaced by a new set of normal coordinates \mathbf{Q} (i.e., $Q_1, \dots, Q_{3\alpha}$) through another set of internal symmetry coordinates \mathbf{S} (i.e., $S_1, \dots, S_{3\alpha}$). It can be written in the matrix form as

$$\mathbf{S} = \mathbf{U} \mathbf{X} \quad (5)$$

in which \mathbf{U} is the unitary matrix which will be given explicitly in later, and the normal coordinate \mathbf{Q} is linearly related to the internal symmetry coordinate \mathbf{S} by a transformation as

$$\mathbf{S} = \mathbf{L} \mathbf{Q} \quad (6)$$

where \mathbf{L} matrix is the transformation coefficients which will be defined lately.

In terms of the forms of matrix \mathbf{S} and its time derivative $\dot{\mathbf{S}}$, the kinetic and potential energies of vibration are

$$2T = \tilde{\mathbf{S}} \mathbf{G}^{-1} \dot{\mathbf{S}} \quad (7)$$

$$2V = \tilde{\mathbf{S}} \mathbf{F} \mathbf{S} \quad (8)$$

where \mathbf{G} matrix may be written in the set of quantities $G_{ii'}$ defined by the equation,

$$G_{it'} = \sum_{\alpha} \frac{1}{m_{\alpha}} X_{t\alpha} X_{t'\alpha} \quad (9)$$

and the \mathbf{G}^{-1} is the inverse matrix of the \mathbf{G} . \mathbf{F} is the force constant matrix which is evaluated from the restoring forces of the deformations.

By substituting Eq. (6) into Eq. (7) and Eq. (8), the kinetic and potential energies are given as

$$2T = \tilde{\mathbf{Q}} \tilde{\mathbf{L}} \mathbf{G}^{-1} \mathbf{L} \mathbf{Q} = \tilde{\mathbf{Q}} \mathbf{E} \dot{\mathbf{Q}} \quad (10)$$

$$2V = \tilde{\mathbf{Q}} \tilde{\mathbf{L}} \mathbf{F} \mathbf{L} \mathbf{Q} = \mathbf{Q} \mathbf{A} \mathbf{Q} \quad (11)$$

where \mathbf{E} is the unit matrix, and \mathbf{A} is a diagonal matrix whose elements have the quantities $\lambda_k = 4\pi^2 \nu_k^2$. From Eq. (10), it follows,

$$\tilde{\mathbf{L}} \mathbf{G}^{-1} \mathbf{L} = \mathbf{E} \quad (12)$$

or

$$\tilde{\mathbf{L}} = \mathbf{L}^{-1} \mathbf{G} \quad (13)$$

and from Eq. (11), it can be also given as

$$\tilde{\mathbf{L}} \mathbf{F} \mathbf{L} = \mathbf{A} \quad (14)$$

Substituting Eq. (13) into Eq. (14), and then multiplying by the \mathbf{L} matrix on the left, we obtain

$$\mathbf{G} \mathbf{F} \mathbf{L} = \mathbf{L} \mathbf{A} \quad (15)$$

This is a set of simultaneous equations written out as

$$\sum_{t'} [(GF)_{tt'} - \delta_{tt'} \lambda_k] L_{t'k} = 0 \quad (16)$$

$$k = 1, 2, \dots, n$$

The transformation coefficients \mathbf{L} matrix are normalized by the relation of Eq. (13) as

$$\mathbf{L} \tilde{\mathbf{L}} = \mathbf{G}. \quad (17)$$

The use of the group theory may remarkably reduce the laborious numerical calculations of Eqs. (16) and (17). Without any other informations, the geometry of the system and its symmetry can offer the number of normal modes of vibration, their degeneracies, and the selection rule for the possible modes of vibration.

An example presented here has a planar trigonal configuration of D_{3h} symmetry. The matrix elements of the symmetry coordinate \mathbf{S} are given by

$$\begin{aligned} S_{A_1'} &= 1/\sqrt{3} (\Delta r_1 + \Delta r_2 + \Delta r_3) \\ S_{A_1'} &= 1/\sqrt{3} (\Delta \theta_1 + \Delta \theta_2 + \Delta \theta_3) \\ S_{E_a'} &= 1/\sqrt{6} (2\Delta r_1 - \Delta r_2 - \Delta r_3) \\ S_{E_a'} &= 1/\sqrt{6} (2\Delta \theta_1 - \Delta \theta_2 - \Delta \theta_3) \\ S_{E_b'} &= 1/\sqrt{2} (\Delta r_2 - \Delta r_3) \\ S_{E_b'} &= 1/\sqrt{2} (\Delta \theta_2 - \Delta \theta_3) \\ S_{A_g} &= \gamma \end{aligned} \quad (18)$$

in which the second equation may be dropped out due to the condition of a redundancy, that

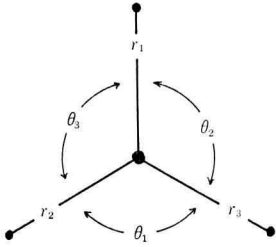


Fig. 2 The geometry of the system.

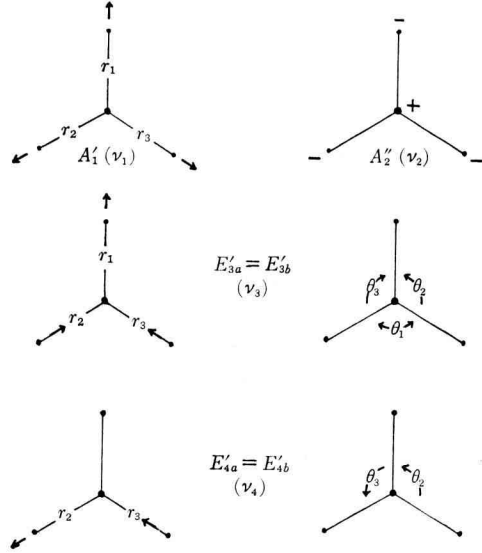


Fig. 3. The normal mode of each type of vibrations.

is, $\Delta\theta_1 + \Delta\theta_2 + \Delta\theta_3$ should be zero, and the last equation may be inserted into equation (18) to fulfil for a closure property thereby γ is a parameter taken arbitrarily. Fig. 2 shows the geometry of the system, in which r_1 , r_2 and r_3 are the equilibrium distances of the mass points and θ_1 , θ_2 and θ_3 are the angles labelled as in the figure. Fig. 3 shows the normal modes of vibration for the present example.

The unitary matrix U for the present case has the form of the square matrix of seven order:

$$U = \begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} & 0 & 0 & 0 & 0 \\ 2/\sqrt{6} & -1/\sqrt{6} & -1/\sqrt{6} & 0 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2/\sqrt{6} & -1/\sqrt{6} & -1/\sqrt{6} & 0 \\ 0 & 0 & 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (19)$$

The matrix elements of G -matrix are given in the form

$$\begin{aligned} G_{A_1'} &= \mu_m \\ G_{A_2'} &= 3(3\mu_M + \mu_m) \\ G_{E'} &= \begin{bmatrix} 3/2\mu_M + \mu_m & 3\sqrt{3}/2\mu_M \\ 3\sqrt{3}/2\mu_M & 3(3/2\mu_M + \mu_m) \end{bmatrix} \end{aligned} \quad (20)$$

where μ is the reduced mass of the hypothetical mass points m and M which are evaluated by the relation between the concentration of stresses and the geometry of the system.

The F -matrix has the same form as in the case of the G -matrix as:

$$\begin{aligned} F_{A_1}' &= F_{11} \\ F_{A_1}'' &= F_{22} \\ F_{E'} &= \begin{bmatrix} F_{33} & F_{34} \\ F_{34} & F_{44} \end{bmatrix} \end{aligned}$$

in which the F_{ii} may be defined to be the force constants for restoring to the original configuration of mass points at equilibrium from the deformed position by vibration of the normal mode itself, and the F_{ij} is the terms of their interactions.

Then, combining Eqs. (5) and (6), the amount of displacements of the internal coordinate can be given by

$$R = \tilde{U} L Q \quad (22)$$

In the present example, the deformations of the mass points may be expressed in the linear combination of normal coordinates as follows:

$$\begin{aligned} \Delta r_1 &= 1/\sqrt{3} L_{11} Q_1 + 2/\sqrt{6} (L_{22} Q_2 + L_{23} Q_3) \\ \Delta r_2 &= 1/\sqrt{3} L_{11} Q_1 - 1/\sqrt{6} (L_{22} Q_2 + L_{23} Q_3) \\ &\quad + 1/\sqrt{2} (L_{22} Q'_2 + L_{23} Q'_3) \\ \Delta r_3 &= 1/\sqrt{3} L_{11} Q_1 - 1/\sqrt{6} (L_{22} Q_2 + L_{23} Q_3) \\ &\quad - 1/\sqrt{2} (L_{22} Q'_2 + L_{23} Q'_3) \end{aligned} \quad (23)$$

4. Coriolis Coupling Coefficients

It is of interest to elucidate a familiar fact that a rotating body at high speed rushes suddenly out in axial direction under certain circumstances. In another case, experiences show an existence of the jumping and dancing movements of a mechanical unit containing an asymmetric rotator at an unsuitable speed of rotation. These phenomena may be given rise to various sources of the motions of translation, vibration and rotation, and their couplings. The present work is intended to make clear the effect of interaction between vibration and rotation, so-called Coriolis coupling⁽¹⁵⁾, which is considered to be one of the sources mentioned above.

Generally, if any displacement vector r_α created by vibration is rotated about a moving axis with angular velocity specified by ω , the direction of the resulting force will be perpendicular to the plane determined by the vector r_α and ω . It may be written as in the last term of Eq. (3),

$$C = \omega \cdot \sum_{\alpha} (m_{\alpha} r_{\alpha} \times v_{\alpha}). \quad (24)$$

From Eq. (1), Eq. (24) gives

$$C = \omega \cdot \sum_{\alpha} [m_{\alpha} (\rho_{\alpha} + a_{\alpha}) \times v_{\alpha}]. \quad (25)$$

The Eckart's condition⁽¹⁶⁾ is introduced in the movements at equilibrium position as follows,

$$\sum_{\alpha} m_{\alpha} a_{\alpha} \times v_{\alpha} = 0. \quad (26)$$

Hence, Eq. (25) is rewritten in the form of the kinetic energy of a vibrating-rotating system as,

$$2T = \omega_x \sum_{\alpha} m_{\alpha} (\rho_{\alpha} \times v_{\alpha})_x + \omega_y \sum_{\alpha} m_{\alpha} (\rho_{\alpha} \times v_{\alpha})_y + \omega_z \sum_{\alpha} m_{\alpha} (\rho_{\alpha} \times v_{\alpha})_z = \mathcal{Q}_x \omega_x + \mathcal{Q}_y \omega_y + \mathcal{Q}_z \omega_z \quad (27)$$

where ω_x , ω_y , and ω_z are the components of the angular velocity of the rotating system. making use of Jahn's rule⁽¹⁸⁾, the \mathcal{Q} 's are connected with the normal coordinates as

$$\mathcal{Q} = \tilde{\mathbf{Q}} \boldsymbol{\zeta} \dot{\mathbf{Q}} \quad (28)$$

It is taken as the definition of the matrix $\boldsymbol{\zeta}$, which is called the Coriolis coupling coefficients. The vectors \mathcal{Q} are the angular momenta of the infinitesimal displacement vectors due to vibration. From the relations between the cartesian coordinate or internal coordinate and the normal coordinate,

$$\boldsymbol{\zeta} = \tilde{\mathbf{L}} \mathbf{C} \tilde{\mathbf{L}}^{-1} \quad (29)$$

in which the elements of \mathbf{C} are given

$$C_{ij} = \sum_{\alpha} \frac{1}{m_{\alpha}} [S_{i\alpha} \times S_{j\alpha}]. \quad (30)$$

The z-axis is taken perpendicular to the plane of symmetry. The \mathbf{C}_x -matrix elements may be represented as follows,

$$\begin{matrix} & S_{3a} & S_{4a} & S_{3b} & S_{4b} \\ \begin{matrix} S_{3a} \\ S_{4a} \\ S_{3b} \\ S_{4b} \end{matrix} & \begin{pmatrix} 0 & 0 & C_{33}^z & C_{34}^z \\ 0 & 0 & C_{34}^z & C_{44}^z \\ -C_{33}^z & -C_{34}^z & 0 & 0 \\ -C_{34}^z & -C_{44}^z & 0 & 0 \end{pmatrix} \end{matrix} \quad (31)$$

The required S-vectors are given explicitly by the following expressions in terms of cartesian displacements:

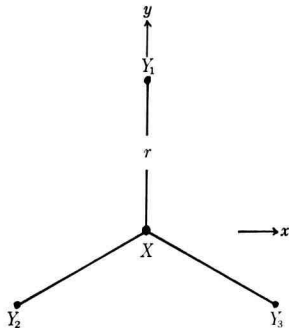


Fig. 4. The cartesian coordinate of the system which is used for calculation of Coriolis coupling constant.

$$\begin{array}{lcl} & x & y & z \\ X & (0 & 0 & 0) \\ Y_1 & (0 & r & 0) \\ Y_2 & (-\frac{3}{2}r & -\frac{1}{2}r & 0) \\ Y_3 & (\frac{3}{2}r & -\frac{1}{2}r & 0) \end{array}$$

$$\begin{aligned}
S_{3a} &= \frac{\sqrt{2}}{2} \left[-3^{1/2} y_1 + \frac{2}{3} 3^{1/2} y_2 + \frac{1}{2} x_3 + \frac{1}{6} 3^{1/2} y_3 - \frac{1}{2} x_4 + \frac{1}{6} 3^{1/2} y_4 \right] \\
S_{4a} &= \frac{\sqrt{2}}{2} \left[-3 y_1 - \frac{1}{2} 3^{1/2} x_3 + \frac{3}{2} y_3 + \frac{1}{2} 3^{1/2} x_4 + \frac{3}{2} y_4 \right] \\
S_{3b} &= \frac{\sqrt{2}}{2} \left[3^{1/2} x_1 - \frac{1}{2} 3^{1/2} x_3 - \frac{1}{2} y_3 - \frac{1}{2} 3^{1/2} x_4 + \frac{1}{2} y_4 \right] \\
S_{4b} &= \frac{\sqrt{2}}{2} \left[3 x_1 - 2 x_2 - \frac{1}{2} x_3 + \frac{1}{2} 3^{1/2} y_3 - \frac{1}{2} x_4 - \frac{1}{2} 3^{1/2} y_4 \right]
\end{aligned} \tag{32}$$

Fig. 4 shows the geometry used in the calculation of the C -matrix. The result obtained for the C^z -matrix is given below:

$$C^z = \begin{pmatrix} \frac{3}{2} \mu_M & 3^{1/2} \left(\frac{3}{2} \mu_M + \mu_m \right) \\ 3^{1/2} \left(\frac{3}{2} \mu_M + \mu_m \right) & \frac{9}{2} \mu_M \end{pmatrix} \tag{33}$$

To show the general relation⁽¹⁹⁾ for the C and G matrices, we get from Eqs. (20) and (33)

$$G^{-1} = \begin{vmatrix} \frac{\frac{3}{2} \mu_M + \mu_m}{\mu_m (3 \mu_M + \mu_m)} & -\frac{\frac{1}{2} 3^{1/2} \mu_M}{\mu_m (3 \mu_M + \mu_m)} \\ -\frac{\frac{1}{2} 3^{1/2} \mu_M}{\mu_m (3 \mu_M + \mu_m)} & \frac{\frac{3}{2} \mu_M + \mu_m}{3 \mu_m (3 \mu_M + \mu_m)} \end{vmatrix} \tag{34}$$

and

$$G^{-1} C = \begin{vmatrix} 0 & 3^{1/2} \\ 3^{-1/2} & 0 \end{vmatrix} \tag{35}$$

The C -matrix is symmetrical and the $G^{-1}C$ -matrix has skew-symmetrical form.

5. Discussions

It is of interest to introduce the method which is a powerful tool to solve the molecular dynamics into the analysis of vibration in the field of engineering. An unimaginable rupture of a thick shaft of rotators at high speed is frequently experienced in engines, such as supercharger. It may be considered to be of the accumulation of energy among the crystal lattices or the grain boundary of polycrystalline metals. The energy could be absorbed by the lattice vibration are several of Mega Hz in frequency.⁽¹²⁾ The rupture of engineering material may be happened to prevail the energies accumulated strains brought about external forces over those dispersed into crystal lattices by their relaxation. When a solid is set into oscillatory motion, it dissipates its vibrational energy into the surrounding medium. As a result of many investigations, Zener⁽²⁰⁾ has pointed out that the mechanical properties of solid satisfy the following requirements:

1. Stress and strain are not single-valued functions of each other.
2. Stress is linearly related to strain.

The first description shows that the vibrational motion can be expressed by the linear combination of several independent functions which have particular frequencies (e.g., the fundamental frequencies of normal modes of vibration, their overtones and combination-tones). The fundamental frequency of normal modes of vibration is denoted by the similar relation formulated by Landau⁽¹⁰⁾ as

$$\nu = \left[\pi^2 K \left(\frac{l^2}{a^2} + \frac{m^2}{b^2} + \frac{n^2}{c^2} \right) \right]^{1/2} \quad (36)$$

where l , m and n are integers and K is the constant which has a dimension of ($\text{cm}^2 \text{sec}^{-2}$). This expression which has a familiar form in the box-approximation of the quantum theory, shows characteristic frequencies for the vibration of a rectangular parallelepiped elastic solid having edges a , b and c .

The normal coordinate method is a technique of the transformation of the kinetic and potential energies of vibration described by cartesian coordinates to those of normal modes characterized by the symmetry properties. The treatments presented here is restricted to the case of the infinitesimal amplitudes of vibration, since all displacements may be considered to be fairly small and only first-order terms will be evaluated. The magnitude of displacements is taken to be equal to the increment produced by unit displacement of the mass point in the most effective direction. The detailed procedures should be referred to the excellent text by E.B. Wilson et al.⁽¹³⁾.

The kinetic energy of vibration in terms of normal coordinate is expressed by the \mathbf{G} -matrix as shown in Eq. (20). The A'_1 mode of vibration is totally symmetric and the A'_2 mode is perpendicular to the symmetry plane. The doubly degenerate mode (E'_{3a} , E'_{4a}) and (E'_{3b} , E'_{4b}) have the same \mathbf{G} -matrix elements which are two dimensional matrices.

The elements of the \mathbf{F} -matrix have the same form as in the case of the \mathbf{G} -matrix.

The frequency of each mode of normal vibrations can be, in principle, obtained from Eqs. (15) and (16). From Eq. (15), the elements of the \mathbf{L} -matrix can be calculated, and they are normalized by the relation of Eq. (17).

To get a satisfactory solution to withstand the practical applications, the matrix iteration method⁽²¹⁾ is the most efficient. The typical arithmetic operation, namely, the addition of a set of products of the matrix is more useful than some of the computations involved in the methods of calculation with electronic computer. Moreover, this iteration method is the most rapid for determining the required characteristic value, because it can be eliminated even numerical error included in an early stage as the calculation progresses.

In molecular dynamics, lots of informations concerning the planar configuration of D_{3h} symmetry are given in literatures^(22,23). There would be four fundamental mode of vibration of species $A'_1 + A''_2 + 2E'$ in this case. The totally symmetric vibration (species A''_1) is inactive in the infrared spectrum, the asymmetric vibration (species A''_2) is inactive in the Raman spectrum and the two asymmetric vibrations (species E') which is doubly degenerated are active in both the infrared and Raman spectrum. The previous work of present author is related to the vibration analysis on gallium triiodide⁽²⁴⁾, which has the same symmetry.

As is well known, if the motion of a particle is referred to a uniformly rotating coordinate system, apart from the acceleration produced by the acting forces, two additional forces will be appeared: one is the centrifugal and the other is the Coriolis forces. The magnitude of the former force is given by

$$F_{\text{centrifugal}} = mr\omega^2, \quad (37)$$

and the latter by

$$F_{\text{Coriolis}} = 2mv_a\omega\sin\varphi, \quad (38)$$

where m is the mass of particle, v_a its apparent velocity with respect to the moving coordinate system, r the distance from the axis of rotation, ω the angular velocity of the coordinate system with respect to a fixed coordinate system, φ the angle between the axis of rotation and the direction of v_a . The Coriolis force occurs only for a moving particle ($v_a \neq 0$) and directs at right angle to its direction of motion and at right angles to the axis of rotation.

The introduction of the Coriolis force to the many body problem leads to an additional coupling between rotation and vibration which is in general much greater than the effect of the centrifugal force, since the velocity due to vibration (v_a) is usually much larger than that due to rotation ($r\omega$).

The preliminary calculations of the matrix elements are included in this paper. We should continue further work toward this goal.

The author wishes to express his hearty thanks to Director Tatsuo Kamiyo of Mitsubishi Chemical Engineering Comp. Ltd. for his valuable discussions and continual encouragements.

Nomenclatures

- D_{3h} : The group theoretical notation of a point group which has a planar configuration of regular triangle.
- A'_1 : The spectroscopic notation of a totally symmetric mode of vibration which corresponds to the frequency of vibration ν_1 in cm^{-1} .
- A''_2 : The spectroscopic notation of a perpendicular mode of vibration which corresponds to the frequency of vibration ν_2 in cm^{-1} .
- E'_{3a} , E'_{3b} ; E'_{4a} , E'_{4b} : The spectroscopic notations of the degenerate modes of vibration. E_{3a} and E_{3b} correspond to the vibrational frequency ν_3 , and E_{4a} and E_{4b} to that ν_4 in cm^{-1} , respectively.

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