

Transient Response of Quasi-rms Rectifier

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Abstract

The response of quasi-rms rectifier for the step input of sinusoidal signal is analyzed and calculated numerically. The results are compared with the cases for the exponential and true-rms responses, both in rising and falling behavior. The numerical differences in these values are shown, indicating smaller response of the quasi-rms case at the initial stage of rise, compared with the true-rms response.

§ Introduction

In the last paper about half-wave quasi-rms rectifier, published in Ikutoku Kiyo¹⁾, the results of investigation of the approximation for rms value, in the steady state, have been shown varying the ratio of two resistances employed in the rectifier circuits. Because this type of rectifier is employed in the ac to dc conversion circuit for the sound level or vibration measurements, the transient behavior must be made clear. But, until present time, no such investigation has been published and followings are results of our study on this subject.

§ Procedure of analysis.

The typical arrangements of the quasi-rms rectifier are shown Fig. 1 where (a) and (b) represent the full wave circuits and (c) represents the half wave circuit. In these circuits, the ratio of R_1 to R_2 determines the rectifying characteristics. For steady state, it has been shown in the previous paper¹⁾ that the value of the above ratio between 0.1 and 0.25 is favorable from the standpoint to obtain good approximation for the rms value, when half wave circuit is considered. Because the rate of charge flow into condenser C becomes twice in the case of full-wave compared to the half-wave case, the same condenser terminal voltage and the same amount of approximation are obtained when the value of R_2 is halved in full wave circuit. Then the above ratio becomes 0.2 and 0.5.

In the following, the transient characteristic of the condenser voltage V_c for sudden application of sinusoidal input, is obtained for the case of full wave circuit. The rate of increase of V_c is represented by the difference between charging current through R_1 and the discharging current through R_2 . The applied voltage is $E_m \sin t$ and the current through R_1 flows only in the duration when the applied voltage is larger than V_c as indicated by the dotted area in Fig. 2 for the full wave case.

The discharge current is V_c/R_2 , and if it is assumed that V_c stays constant during each charging and discharging period, (assuming that the time constant $C \times (R_1 \cdot R_2 / (R_1 + R_2))$ is

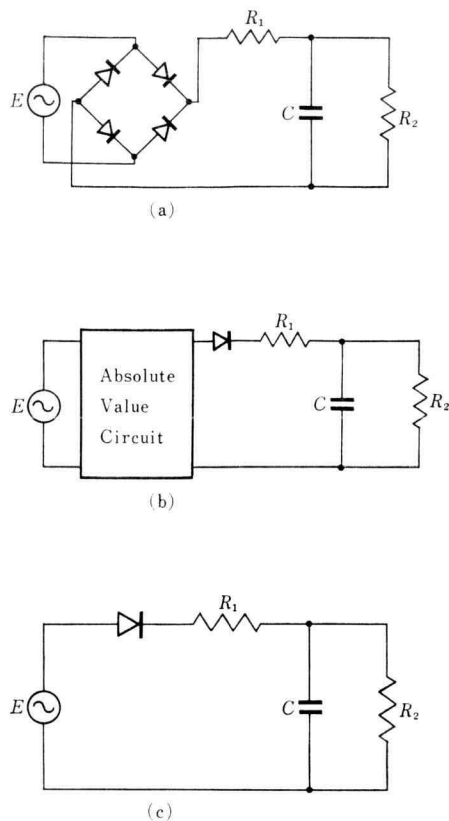


Fig. 1 Quasi-rms rectifier circuits.

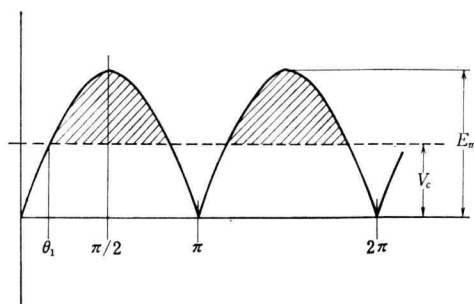


Fig. 2 Charging of condenser

sufficiently large compared to the period of the applied sinusoidal voltage), equation (1) is obtained by taking average for the quarter of a period.

$$C \frac{dV_c}{dt} = \frac{2}{\pi} \left(\frac{1}{R_1} \int_{\theta_1}^{\pi/2} (E_m \sin t - V_c) dt - \frac{V_c}{R_2} \right) \quad (1)$$

where

$$\theta_1 = \sin^{-1} V_c/E_m \quad (2)$$

from Fig. 2. Taking as $v_c = V_c/E_m$, and after integration of the right hand side of equation (1) and employing the relation of equation (2), following expression is obtained.

$$\frac{dv_c}{dt} = \frac{2}{\pi} \frac{1}{CR_1} \sqrt{1 - v_c^2} + v_c \left\{ \frac{2}{\pi} \frac{\sin^{-1} v_c}{CR_1} - \left(\frac{1}{CR_1} + \frac{1}{CR_2} \right) \right\} \quad (3)$$

The integration of this differential equation must be made numerically in the form expressed by equation (4),

$$\int_0^{v_c} \frac{dv_c}{\frac{2}{\pi} \frac{1}{CR_1} \sqrt{1 - v_c^2} + v_c \left\{ \frac{2}{\pi} \frac{\sin^{-1} v_c}{CR_1} - \left(\frac{1}{CR_1} + \frac{1}{CR_2} \right) \right\}} = t. \quad (4)$$

§ Calculations and Results.

In the actual calculation of equation (4), the value of R_1/R_2 is selected as 0.2, 0.3, 0.4 and 0.5. Also R_2 is taken as 25 K Ohms and C as 5 microfarads. In the execution of the integration, the Newton-Cortes method as shown in equation (5), was used.

$$S^{(n)} = \frac{3}{8} h (y_1^{(n)} + 3y_2^{(n)} + 3y_3^{(n)} + y_4^{(n)}) \quad (5)$$

In above, $S^{(n)}$ stands for the approximated integral in the n -th interval of 0.01 step of v_c starting from zero ($h=0.01$) and y is the value of the integrand of equation (4) in this interval. Then the relation between t and v_c can be obtained from the relation

$$t_n = \sum_0^n S^{(n)} \quad (6)$$

for each step of this summation. At some value of v_c less than 1, the denominator of the integrand of equation (4) becomes zero and the integrated value, *i. e.* t , becomes infinite. This v_c can be considered as the final condenser voltage ($v_{c\infty}$) for the step input of sinusoidal signal. The results of the above numerical calculation are plotted in Fig. 3 and the final condenser voltage $v_{c\infty}$ is given in Table 1.

Table 1. Final condenser voltage.

R_1/R_2	0.2	0.3	0.4	0.5
$V_{c\infty} [v]$	0.6455	0.5753	0.5170	0.4728

In the recommendation of IEC²⁾ or in other national standards for the sound level meter, there is a description on the dynamic characteristic and, in this sentence, the maximum response for the single tone burst input of duration 0.2 sec is taken to be as 1dB down from the response for the continuous signal or the final condenser voltage.

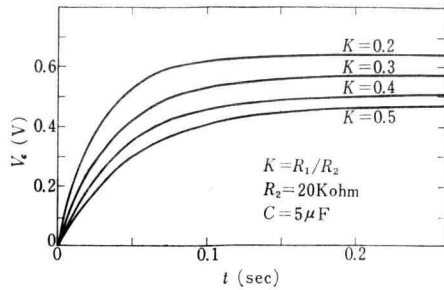


Fig. 3 Charging response of quasi-rms rectifier

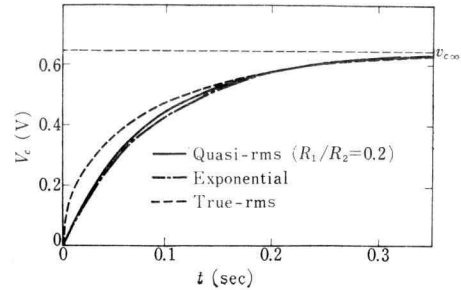


Fig. 4 Comparison of charging response of quasi-rms rectifier exponential and true-rms response.

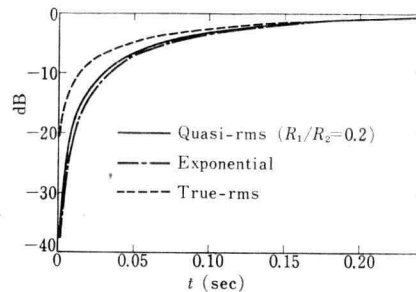


Fig. 5 Comparison of charging response of quasi-rms rectifier, exponential and true-rms responses in dB.

To adjust the curves shown in Fig. 3, for the point corresponding to $t = 0.2$ sec to take -1dB value from the final level, C must be changed to the following values listed in Table 2, instead of the original $5\mu\text{F}$ employed in the calculation.

Table 2. Condenser capacity and time constant for 0.2 sec and -1dB.

R_1/R_2	C [μF]	time const. $C (R_1//R_2)$ [ms]
0.2	15.31	63.80
0.3	11.94	68.87
0.4	10.15	72.49
0.5	9.024	75.20

In this table, $C(R_1//R_2)$ means the calculated time constant in milliseconds for the combination of C and the parallel circuit of R_1 and R_2 . This time constant is nominal value and it does not represent the actual response. The equivalent time constant

for the actual response may be represented by the exponential case discussed in the following.

The adjusted curve, as above, for the case $R_1/R_2=0.2$ is plotted in Fig. 4 by full line. The ordinary exponential transient rise of a condenser terminal voltage, charged by the step input through a resistance, is expressed by equation (7),

$$f_{exp}(t) = 1 - e^{-t/\tau_1} \quad (7)$$

in which τ_1 takes the value 90.14 ms to pass the point 0.2 s and -1dB. This is plotted by chain line in Fig. 4.

If it is desired to indicate true-rms value, instead of quasi-rms, the transient response for the step input of sinusoidal signal must be indicated by equation (8),

$$f_{rms}(t) = \sqrt{1 - e^{-t/\tau_2}} \quad (8)$$

because the square-root operation is made after the exponential rise of the input power. In this expression τ_2 must be chosen as 126.5 ms for the same condition as before. The curve representing equation (8) is also plotted in Fig. 4 by dotted line. In Fig. 5, the same relations, *i.e.* the calculated results of equation (6) for the ratio R_1/R_2 is 0.2, and the curves for equation (7) and (8) are plotted in terms of dB. From Fig. 4 and Fig. 5, it is known that the difference between quasi-rms and exponential cases is not large, but there are considerable differences between quasi-rms and true-rms cases in the initial stages of the rising response. In Table 3 the above dB values are listed.

Table 3. Initial Charging responses.

t [ms]	2	5	10	50	100	200
quasi-rms ($R_1/R_2=0.2$) [dB]	-31.92	-24.16	-18.46	-6.827	-3.338	-1.0
exponential [dB]	-33.29	-25.36	-14.02	-7.417	-3.476	-1.0
true-rms [dB]	-18.04	-14.12	-8.347	-4.860	-2.624	-1.0

The falling responses from some charged terminal voltage of the condenser, are expressed simply by the exponential decaying, $e^{-t/\tau}$ with the time constants τ indicated in Table 4 for all cases discussed above.

Table 4. Falling time constants.

R_1/R_2	Falling time const. τ [ms]
0.2	382.77
0.3	298.4
0.4	253.7
0.5	225.6
exponential	90.14 ($=\tau_1$)
true-rms	253.0 ($=2\tau_2$)

In Table 4, the falling time constants for the quasi-rms circuit are CR_2 , for the exponential case it is τ_1 , and for the true-rms case it is $2\tau_2$. Fig. 6 shows the falling responses from the final value for the above three cases. ($R_1/R_2 = 0.2$ for quasi-rms curve).

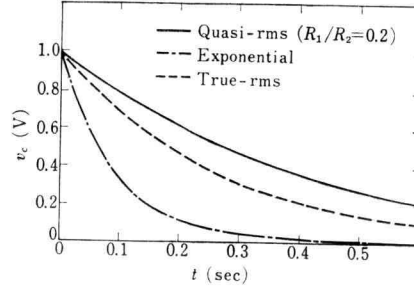


Fig. 6 Comparison of discharging response of quasi-rms rectifier, exponential and true-rms responses.

It must be noticed that, for the quasi-rms case, the falling time constant is large compared to the equivalent rising time constant for the rising response. In the exponential case, rise and fall time constants take the same value.

§ Indicating Characteristic of Meter.

The dynamic response of a moving coil dc meter is represented by the transfer function $T(s)$ between the indicated value and the electrical input, as equation (9)

$$T_1(s) = \frac{\omega_0^2}{s^2 + 2\xi\omega_0 s + \omega_0^2}, \quad (9)$$

where s , ω_0 and ξ are the complex frequency, the natural frequency of the moving part and the damping factor respectively. When the input is represented by the exponential rise or fall with the same time constant τ as discussed in the previous section, the total transfer function becomes

$$T_2(s) = \frac{\omega_0^2}{(1 + \tau)(s^2 + 2\xi\omega_0 s + \omega_0^2)}. \quad (10)$$

The actual response for step input or other inputs, which are Laplace-transformable, can be calculated using the tables of Laplace transforms.

But for other cases, when the Laplace transformation can not be applied, the transient calculation including meter must be made numerically or by means of analogue simulation. These are left for the later work.

§ Conclusions

The behavior of transient response of the quasi-rms rectifier for the step input of sinusoidal signal has been analyzed and numerical integration of the differential equation

which appears in the analysis has been made. As the result, it is known that the smaller initial response of the quasi-rms case compared to the true-rms response is the essential characteristics of this circuit. The final criterion of the response must be made taking into account of the human sensitivity response when the instruments are employed to measure sound or vibration.

§ References

- 1) M. Oshima: "Half-wave Quasi-root-mean-square Rectifier" Ikutoku Research Report, **1**, p. 19 (1973).
- 2) IEC Publication: 123, 179, 179A.