

# Exchange Splitting of ${}^3T_{2g}$ Band in Antiferromagnetic $\text{KNiF}_3$

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## Abstract

The absorption spectra of the  ${}^3T_{2g}$  band in pure  $\text{KNiF}_3$ ,  $\text{KMgF}_3:0.015\text{Ni}$  and layered  $\text{K}_2\text{NiF}_4$  crystals are measured. The stress effect of the  ${}^3T_{2g}$  band in  $\text{KNiF}_3$  is also measured. The stress effect shows that the 6790 and 6951  $\text{cm}^{-1}$  lines both split into two lines. This splitting can be explained qualitatively by the exchange effect in the  ${}^3T_{2g}$  state.

## 1. Introduction

The crystal,  $\text{KNiF}_3$  has many absorption bands in the region of photon energy ranging from 6000 to 50000  $\text{cm}^{-1}$ . The gross features of these absorption bands are now well understood as transitions between levels of a  $\text{Ni}^{2+}$  ion which split by octahedral crystal-field.<sup>1,2)</sup> This crystal is antiferromagnet. Below the Néel temperature, energy levels of a  $\text{Ni}^{2+}$  ion are modified by exchange interaction.<sup>3)</sup> In the  ${}^1E_g^a$  band located at 16000  $\text{cm}^{-1}$ , new absorption bands such as exciton-magnon and exciton-magnon-phonon transitions which are not observed above the Néel temperature, appear in the antiferromagnetic phase.<sup>4)</sup> These new absorption bands follow from the splitting of the ground state,  ${}^3A_{2g}$  by the exchange interaction. Excited states should also split by the exchange interaction. This effect, however, has not been reported.

The purpose of this paper is to study the exchange effect in excited states.  $\text{KNiF}_3$  has three spin-triplet bands, such as  ${}^3T_{2g}$ ,  ${}^3T_{1g}^a$  and  ${}^3T_{1g}^b$  bands. The  ${}^3T_{2g}$  band is mainly a pure electronic transition, *i.e.* a magnetic dipole transition, while the other bands are complicated by phonons.<sup>2)</sup> We study the absorption spectrum of the  ${}^3T_{2g}$  band in pure  $\text{KNiF}_3$  and  $\text{KMgF}_3:0.015\text{Ni}$  crystals to see the exchange effect. The stress effect of  $\text{KNiF}_3$  is studied to examine the fine structures of the  ${}^3T_{2g}$  band. We also study the polarization dependence of absorption spectra of the layered  $\text{K}_2\text{NiF}_4$  to clarify the mechanism of the transition.

## 2. Crystal Structure

Figure 1-(a) shows the crystal structure of the cubic perovskite  $\text{KNiF}_3$ . A  $\text{Ni}^{2+}$  ion locates at the body centered position of the cubic cell. Six  $\text{F}^-$  ions which locate at the face centered positions, form a regular octahedron around the  $\text{Ni}^{2+}$  ion.<sup>5)</sup> The Néel temperature is 246K.<sup>6)</sup> Below this temperature,  $\text{Ni}^{2+}$  ions couple antiferromagnetically through  $\text{F}^-$  ions, with spins parallel to the crystallographic axis.<sup>7)</sup>

Figure 1-(b) shows the structure of the layered  $\text{K}_2\text{NiF}_4$ . An octahedron, formed by six  $\text{F}^-$  ions around the  $\text{Ni}^{2+}$  ion is compressed (1.5%) along the c-axis. The Néel temperature is 97.1 K.<sup>8)</sup> The perovskitelike layers of  $\text{KNiF}_3$  are separated by planes of  $\text{K}^+\text{F}^-$ .  $\text{K}_2\text{NiF}_4$

behaves like a two-dimensional antiferromagnet.

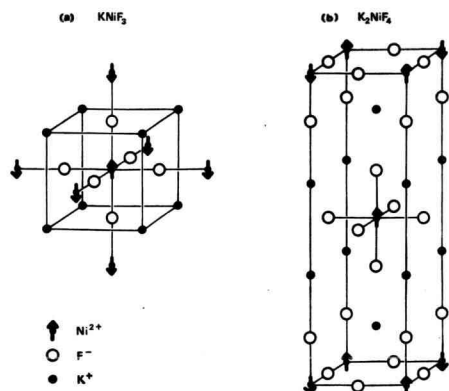


Fig. 1 Crystal structures of (a)  $\text{KNiF}_3$  and (b)  $\text{K}_2\text{NiF}_4$  in the antiferromagnetic phase.

### 3. Experimental

The crystals,  $\text{KMgF}_3:0.015\text{Ni}$  and  $\text{K}_2\text{NiF}_4$  were grown by the flux method.<sup>9,10)</sup> The absorption spectra were measured with a Cary Model 14 spectrophotometer. The stress was applied to a sample by weight through a rod. A Glan-Thompson prism was used as a polarizer.

### 4. Experimental Results

Figure 2 shows the absorption spectra of the  $^3\text{T}_{2g}$  band in  $\text{KNiF}_3$  (a solid curve) and  $\text{KMgF}_3:0.015\text{Ni}$  (a dashed curve) at 77K. There are many structures in both spectra. These peaks are listed in Table I. The separation from the lowest frequency line is also shown in Table I. With shift of  $\text{KMgF}_3:0.015\text{Ni}$  spectrum to higher frequency by  $100\text{cm}^{-1}$ , the frequencies of the lines of both spectra agree well. This suggests that the exchange effect in the  $^3\text{T}_{2g}$  state is not so large as to change throughly the spectrum.

Figure 3 shows the absorption spectra of  $\text{KNiF}_3$  under the  $\langle 001 \rangle$  stress at 77K. The stress

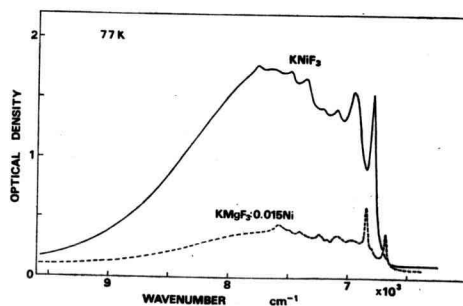
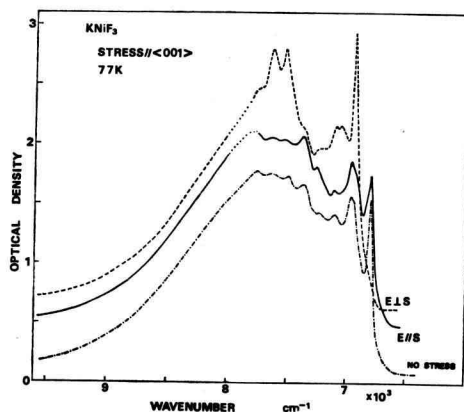


Fig. 2 Absorption spectra of the  $^3\text{T}_{2g}$  band in  $\text{KNiF}_3$  (thickness=0.58 mm) and  $\text{KMgF}_3:0.015\text{Ni}$  (thickness=5.8 mm) at 77K.

Table I. The peaks of the  ${}^3T_{2g}$  band in  $\text{KNiF}_3$  and  $\text{KMgF}_3:0.015\text{Ni}$  at 77K.

$\text{KNiF}_3$		$\text{KMgF}_3: 0.015\text{Ni}$		assignment by Ferguson <sup>2)</sup>
peak position	separation from 6790 $\text{cm}^{-1}$ line	peak position	separation from 6687 $\text{cm}^{-1}$ line	
6790 $\pm$ 2	0	6687 $\pm$ 3	0	$\Gamma_3$
6951 $\pm$ 3	161	6852 $\pm$ 3	165	$\Gamma_4$
7100 $\pm$ 7	310	6960 $\pm$ 5	273	
7228 $\pm$ 6	438	7083 $\pm$ 8	376	
		7167 $\pm$ 5	480	
7353 $\pm$ 8	563	7240 $\pm$ 5	553	$\Gamma_5$
7496 $\pm$ 9	706	7401 $\pm$ 5	714	$\Gamma_2$
7634 $\pm$ 10	844	7499 $\pm$ 8	812	
7755 $\pm$ 10	965	7579 $\pm$ 6	892	

Fig. 3 Absorption spectra of the  ${}^3T_{2g}$  band in  $\text{KNiF}_3$  (thickness=0.58mm) under the  $\langle 001 \rangle$  stress at 77K. Stress is 20MPa.Table II. The peaks of the  ${}^3T_{2g}$  band in  $\text{KNiF}_3$  under the  $\langle 001 \rangle$  and  $\langle 111 \rangle$  stress at 77K.

no stress	stress// $\langle 001 \rangle$		stress// $\langle 111 \rangle$	
	E//S	$E_{\perp}S$	E//S	$E_{\perp}S$
6790 $\pm$ 2	6791 $\pm$ 3		6793 $\pm$ 2	6810 $\pm$ 2
6951 $\pm$ 3	6957 $\pm$ 4	6928 $\pm$ 4	6951 $\pm$ 3	6953 $\pm$ 4
7100 $\pm$ 7	7105 $\pm$ 10	7054 $\pm$ 7 7087 $\pm$ 7	7104 $\pm$ 6	7117 $\pm$ 10
7228 $\pm$ 6	7250 $\pm$ 5	7220 $\pm$ 15	7234 $\pm$ 5	7240 $\pm$ 5
7353 $\pm$ 8	7366 $\pm$ 10		7366 $\pm$ 10	7359 $\pm$ 5
7496 $\pm$ 9	7515 $\pm$ 15	7491 $\pm$ 7	7526 $\pm$ 14	7505 $\pm$ 14
7634 $\pm$ 10	7634 $\pm$ 15	7619 $\pm$ 14	7634 $\pm$ 15	7622 $\pm$ 8

is 20MPa. A solid curve is the spectrum for light polarized parallel to the stress (abbreviated as E//S), and a dashed curve is the spectrum for light polarized perpendicular to the stress ( $E_{\perp}S$ ). A chained curve is the spectrum with no stress. The characteristic properties of these spectra are as follows. The  $6790\text{ cm}^{-1}$  line is not observed for  $E_{\perp}S$ . The  $6951\text{ cm}^{-1}$  line consists of two lines located at  $6928$  and  $6957\text{ cm}^{-1}$ . The  $6928\text{ cm}^{-1}$  line becomes strong for  $E_{\perp}S$ . The lines at  $7100$ ,  $7491$  and  $7619\text{ cm}^{-1}$  become strong for  $E_{\perp}S$ . These peaks are summarized in Table II.

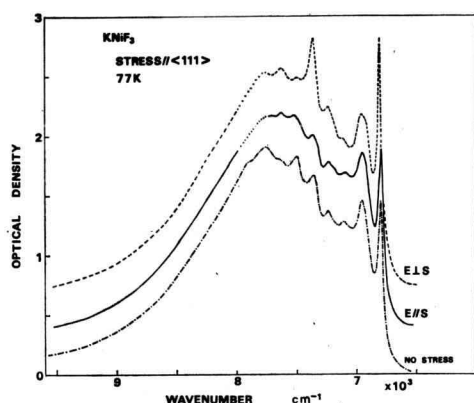


Fig. 4 Absorption spectra of the  ${}^3T_{2g}$  band in  $\text{KNiF}_3$  (thickness=0.62 mm) under the  $\langle 111 \rangle$  stress at 77K. Stress is 15MPa.

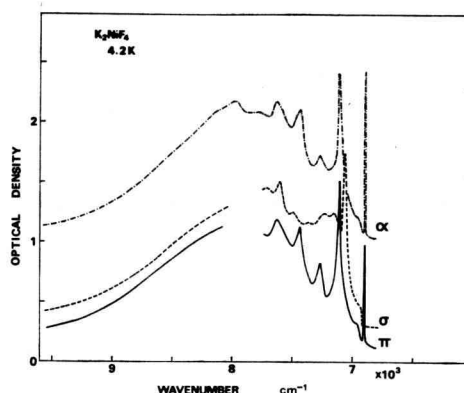


Fig. 5 Absorption spectra of the  ${}^3T_{2g}$  band in  $\text{K}_2\text{NiF}_4$  at 4.2K.

Table III. The peaks of the  ${}^3T_{2g}$  band in  $\text{K}_2\text{NiF}_4$  at 4.2K, and the mechanism of the transition assigned from the polarization dependence of the line. M1 represents a magnetic dipole transition and E1 an electric dipole transition.

$\alpha$ and $\pi$ spectrum	$\sigma$ spectrum	separation from $6895\text{ cm}^{-1}$ line	mechanism of transition
$6895 \pm 3$		0	M1
	$7066 \pm 3$	171	M1
$7110 \pm 3$		215	M1
	$7156 \pm 8$	261	M1
	$7230 \pm 10$	335	} M1 + E1
$7270 \pm 6$		375	
	$7367 \pm 6$	472	M1
$7435 \pm 6$		540	M1
	$7491 \pm 10$	596	M1
	$7605 \pm 4$	710	M1 + E1
$7635 \pm 6$		740	M1
$7800 \pm 10$	—	905	
$7980 \pm 8$	—	1085	

Figure 4 shows the absorption spectra of  $\text{KNiF}_3$  under the  $\langle 111 \rangle$  stress at 77K. The stress is 15MPa. A solid curve is the spectrum for  $E//S$ , a dashed curve is for  $E_{\perp}S$  and a chained curve is with no stress. These spectra show following results. The  $6790\text{ cm}^{-1}$  line consists of two lines located at  $6793$  and  $6810\text{ cm}^{-1}$ . The  $6793\text{ cm}^{-1}$  line is observed for  $E//S$ , and the  $6810\text{ cm}^{-1}$  line becomes strong for  $E_{\perp}S$ . The  $7364\text{ cm}^{-1}$  line also becomes strong for  $E_{\perp}S$ . These peaks are listed in Table II. The effect of the  $\langle 001 \rangle$  and  $\langle 111 \rangle$  stress shows that the  $6790$  and  $6951\text{ cm}^{-1}$  lines both consist of two lines.

Figure 5 shows the absorption spectra of  $\text{K}_2\text{NiF}_4$  at 4.2K. The  $\alpha$  spectrum (a chained curve) is for light propagated parallel to the  $c$ -axis of the crystal. The  $\pi$  (a solid curve) and  $\sigma$  (a dashed curve) spectra are for light polarized parallel and perpendicular to the  $c$ -axis. The frequencies of the peaks and the mechanism of the transition assigned from the polarization dependence are listed in Table III. In the Table, M1 represents a magnetic dipole transition and E1 an electric dipole transition.

## 5. Discussion

In this section, we try to assign the observed lines of the  $^3T_{2g}$  band. The  $^3T_{2g}$  state splits into four states such as  $\Gamma_2$ ,  $\Gamma_3$ ,  $\Gamma_4$  and  $\Gamma_5$  states by spin-orbit interaction. Liehr *et al.* calculated levels of a  $\text{Ni}^{2+}$  ion, using a four-parameter crystal-field approach.<sup>11)</sup> Ferguson *et al.* decided the four parameters by fitting to lines in  $\text{KMgF}_3:\text{Ni}$  spectrum.<sup>2)</sup> They assigned for the  $^3T_{2g}$  band,  $6790\text{ cm}^{-1}$  line as  $\Gamma_3$ ,  $6951\text{ cm}^{-1}$  line as  $\Gamma_4$ ,  $7353\text{ cm}^{-1}$  line as  $\Gamma_5$  and  $7496\text{ cm}^{-1}$  line as  $\Gamma_2$ , as shown in Table I. Hereafter we analyze these lines following their assignment.

Comparison of the spectrum of  $\text{KNiF}_3$  with those of  $\text{K}_2\text{NiF}_4$  shows that  $6790\text{ cm}^{-1}$  line in  $\text{KNiF}_3$  corresponds to  $6895\text{ cm}^{-1}$  line in  $\text{K}_2\text{NiF}_4$ ,  $6951\text{ cm}^{-1}$  line to  $7110\text{ cm}^{-1}$  line,  $7353\text{ cm}^{-1}$  line to  $7435\text{ cm}^{-1}$  line and  $7496\text{ cm}^{-1}$  line to  $7635\text{ cm}^{-1}$  line. These lines in  $\text{K}_2\text{NiF}_4$  are magnetic dipole transitions. This shows that the corresponding lines of  $\text{KNiF}_3$  are also magnetic dipole transitions.

We consider the stress effect of  $\text{KNiF}_3$ . These extremely anisotropic absorption spectra are caused by change of magnetic domains under stress, as described in the previous paper.<sup>4)</sup> In the absence of stress,  $\text{KNiF}_3$  has antiferromagnetic domains and the direction of and collinear spins distributes randomly among  $\langle 001 \rangle$ ,  $\langle 100 \rangle$  and  $\langle 010 \rangle$  directions. When the stress is applied, the crystal turns to a single domain, with spins parallel to the stress. The critical stress is 3MPa for the  $\langle 001 \rangle$  direction, and 6MPa for the  $\langle 111 \rangle$  direction. The ground state,  $^3A_{2g}$  splits by exchange interaction into three levels apart by about  $330\text{ cm}^{-1}$ ,<sup>2)</sup> and at the low temperature the transitions occur from the lowest level. These transitions are anisotropic for the direction of spins.

The stress effect further shows that  $6790$  and  $6951\text{ cm}^{-1}$  lines both consist of two lines. In the following, we discuss the effect of this splitting. The X-ray measurement shows that the crystal structure of  $\text{KNiF}_3$  is cubic, even in the antiferromagnetic phase.<sup>9)</sup> There is, however, the possibility that the lattice is slightly deformed along the direction of spins. We assume that this is the case. Under the  $\langle 001 \rangle$  stress, a  $\text{Ni}^{2+}$  ion is affected by the tetragonal

crystal-field. The  $\Gamma_3$  and  $\Gamma_4$  states both split into two levels. Under the  $\langle 111 \rangle$  stress, a  $\text{Ni}^{2+}$  ion is affected by the trigonal crystal-field. The  $\Gamma_3$  state does not split in this case. This does not agree with the observed result. Even if a  $\text{Ni}^{2+}$  ion is assumed to be affected by the trigonal crystal-field in addition to the tetragonal crystal-field, the split components of the  $\Gamma_3$  state have equal intensities for E//S and  $E_\perp$ S polarizations. This result is contrary to the fact that the  $6793 \text{ cm}^{-1}$  line is observed for E//S and the  $6810 \text{ cm}^{-1}$  line is for  $E_\perp$ S. We consider whether the  $6790 \text{ cm}^{-1}$  line is really the  $\Gamma_3$  state. The  $6790 \text{ cm}^{-1}$  line is observed for E//S, and not observed for  $E_\perp$ S under the  $\langle 001 \rangle$  stress. Among the levels,  $\Gamma_2$  and  $\Gamma_3$  states have this polarization dependence. The  $\Gamma_2$  state is singlet and splits no further. Thus the  $6790 \text{ cm}^{-1}$  line is confirmed as the  $\Gamma_3$  state. The other possibility is that the ground state splits into levels apart by about 20 to  $30 \text{ cm}^{-1}$ , under the crystal-field of low symmetry.<sup>12)</sup> This effect, however, does not occur in  $\text{KNiF}_3$ , as the exchange interaction is dominant in the ground state. The observed splitting cannot be explained by the effect of crystal-field of low symmetry.

With respect to the effect of the lattice vibrations, the normal modes of  $\text{KNiF}_3$  are  $4T_{1u} + T_{2u}$ . They are all odd parity modes. The Jahn-Teller effect does not occur.

We consider the exchange interaction in the excited state,  ${}^3T_{2g}$ . The Hamiltonian for the exchange interaction in the  ${}^3T_{2g}$  state,  $\mathcal{H}_{ex}$  is represented by the molecular field approximation as,

$$\mathcal{H}_{ex}' = g \beta H_{ex} S_z \quad (1)$$

where  $g$  is Landé  $g$ -factor,  $\beta$  is Bohr magneton,  $H_{ex}$  is the exchange field applied to the  ${}^3T_{2g}$  state and  $S_z$  is the  $z$  component of spin. Ferguson *et al.* decided the four parameters as  $Dq = 698 \text{ cm}^{-1}$ ,  $F_2 = 1520 \text{ cm}^{-1}$ ,  $F_4 = 114 \text{ cm}^{-1}$  and  $\lambda = -310 \text{ cm}^{-1}$  by fitting to the levels of the  $\text{KMgF}_3$ : Ni spectrum. Their calculated values of the  ${}^3T_{2g}$  band are as follows,  $\Gamma_3$ :  $6713 \text{ cm}^{-1}$ ,  $\Gamma_4$ :  $6863 \text{ cm}^{-1}$ ,  $\Gamma_5$ :  $7265 \text{ cm}^{-1}$  and  $\Gamma_2$ :  $7412 \text{ cm}^{-1}$ . These values shift to lower frequency by about  $80 \text{ cm}^{-1}$ , compared with the observed frequencies of  $\text{KNiF}_3$ . For the moment, we use these values and calculate the energy levels including the exchange interaction of eq. (1).

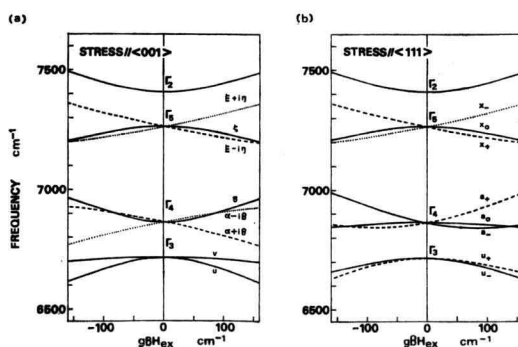


Fig. 6 The calculated result of the energy levels of the  ${}^3T_{2g}$  state versus  $g\beta H_{ex}$ , under (a) the  $\langle 001 \rangle$  stress, (b) the  $\langle 111 \rangle$  stress. A solid curve is allowed for E//S, a dashed curve for  $E_\perp$ S, and a dotted curve is forbidden.

Table IV. The calculated result of transition intensities,  $I_{E//S}$  for E//S and  $I_{E\perp S}$  for E $\perp$ S, under the  $\langle 001 \rangle$  and  $\langle 111 \rangle$  stress.  $I_{NS}$  is intensity with no stress.

level	$I_{NS}$	$\langle 001 \rangle$ stress					$\langle 111 \rangle$ stress				
		state	$I_{E//S}$	$I_{E\perp S}$	$\frac{I_{E//S}}{I_{NS}}$	$\frac{I_{E\perp S}}{I_{NS}}$	state	$I_{E//S}$	$I_{E\perp S}$	$\frac{I_{E//S}}{I_{NS}}$	$\frac{I_{E\perp S}}{I_{NS}}$
$\Gamma_3$	2/9	u	1/4	0	3/2	0	u-	1/6	0	3/4	0
		v	1/12	0			u+	0	1/3	0	3/2
$\Gamma_4$	1/3	$\alpha+i\beta$	0	1/2	0	3/2	a-	1/3	0	5/4	0
		$\gamma$	1/4	0	3/4	0	a <sub>0</sub>	1/12	0		
		$\alpha-i\beta$	0	0	0	0	a+	0	1/6	0	1/2
$\Gamma_5$	1/3	$\xi-i\eta$	0	1/2	0	3/2	x+	0	1/2	0	3/2
		$\zeta$	1/4	0	3/4	0	x <sub>0</sub>	1/4	0	3/4	0
		$\xi+i\eta$	0	0	0	0	x-	0	0	0	0
$\Gamma_2$	1/9	e <sub>2</sub>	1/6	0	3/2	0	e <sub>2</sub>	1/6	0	3/2	0

Figure 6-(a), (b) show the eigenvalues of the  ${}^3T_{2g}$  state versus  $g\beta H_{ex}$  under the  $\langle 001 \rangle$  and  $\langle 111 \rangle$  stress. A solid curve shows that the transition to this level is allowed for E//S, and a dashed curve for E $\perp$ S. A dotted curve shows that the transition to this level is forbidden. Their relative intensities are listed in Table IV. In this calculation, we consider the transition from the  $S_z = -1$  site of the ground state. For the transition from the  $S_z = 1$  site, the sign of  $g\beta H_{ex}$  is changed and Table IV is rewritten as follows:  $\alpha+i\beta$  state is exchanged with  $\alpha-i\beta$  state,  $\xi+i\eta$  state with  $\xi-i\eta$  state, u<sub>+</sub> state with u<sub>-</sub> state, a<sub>+</sub> state with a<sub>-</sub> state and x<sub>+</sub> state with x<sub>-</sub> state. In Table IV,  $I_{NS}$  is the intensity with no stress, where spins are oriented randomly among the three equivalent directions,  $\langle 001 \rangle$ ,  $\langle 100 \rangle$  and  $\langle 010 \rangle$ .  $I_{E//S}$  and  $I_{E\perp S}$  are intensities for E//S and E $\perp$ S.

We compare them with the observed results. The sign of  $g\beta H_{ex}$  is positive to fit to the observed result. Under the  $\langle 001 \rangle$  stress, the  $\Gamma_3$  state becomes strong (by factor of 3/2) for E//S, and is forbidden for E $\perp$ S. Under the  $\langle 111 \rangle$  stress, the u<sub>-</sub> state becomes weak (3/4) for E//S, and the u<sub>+</sub> state becomes strong (3/2) for E $\perp$ S. This result agrees with the 6790 cm<sup>-1</sup> line. For the  $\Gamma_4$  state under the  $\langle 001 \rangle$  stress, the  $\gamma$  state becomes weak (3/4) for E//S, the  $\alpha+i\beta$  state becomes strong (3/2) for E $\perp$ S. Under the  $\langle 111 \rangle$  stress, the a<sub>0</sub> and a<sub>-</sub> states become strong (5/4) for E//S, and the a<sub>+</sub> state becomes weak (1/2) for E $\perp$ S. For the  $\langle 001 \rangle$  stress, this result agrees with the 6951 cm<sup>-1</sup> line. For the  $\langle 111 \rangle$  stress, however, the 6951 cm<sup>-1</sup> line does not show structure. For the  $\Gamma_5$  state, the calculated result agrees with the 7353 cm<sup>-1</sup> line, except that this line is not so strong for E $\perp$ S under the  $\langle 001 \rangle$  stress. The 7496 cm<sup>-1</sup> line does not agree with the calculated result of the  $\Gamma_2$  state. The exchange interaction in the  ${}^3T_{2g}$  state can explain the gross feature of the structures, especially the splitting and polarization dependence of the 6790 and 6951 cm<sup>-1</sup> lines. There remains the problem to analyze quantitatively the levels and to obtain the amplitude of the exchange interaction in the  ${}^3T_{2g}$  state.

## 6. Conclusion

The absorption spectra of the  ${}^3T_{2g}$  band in pure  $\text{KNiF}_3$  and  $\text{KMgF}_3:0.015 \text{ Ni}$  crystals are measured to see the exchange effect in the excited  ${}^3T_{2g}$  state. The exchange effect in the  ${}^3T_{2g}$  state is not so large as to change throughly the spectrum. The polarization dependence of the absorption spectra of layered  $\text{K}_2\text{NiF}_4$  is measured to clarify the mechanism of the transition. The stress effect of  $\text{KNiF}_3$  is studied to examine the structures of the  ${}^3T_{2g}$  band. The stress effect shows that the 6790 and 6951  $\text{cm}^{-1}$  lines both split into two lines. This splitting can be explained qualitatively by the exchange effect in the  ${}^3T_{2g}$  state.

## Acknowledgements

The author would like to express his sincere thanks to Professor G. Kuwabara for allowing him to use the experimental facilities at the university of Tokyo. He also thanks to Dr. A. Tsuchida for providing him the  $\text{KNiF}_3$  samples.

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