On the Helical Vortex in an Agitating Tank.

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Abstract

A vortex produced in an agitating tank is the most familiar properties of fluids. It is well known that the steady-state flow of fluids decreases the efficiency of agitation. Usual experiences talk us how to get the turbulent flow for our purpose. However, various kinds of vortices which are in our experiences have not yet given us the sufficient knowledges. An approach to solve the mechanism of formation of the vortex is proposed preleminarily here. The vortex is the characteristic natures of the boundary surface of fluid. The shapes of vortices are greatly affected with the movement of fluids as a whole, such as the rotatory and laminar flow of the fluid in an agitating tanks. One type of vortex which is helical shaped one is discussed in some details.

Introduction

Two types of vortex are found in nature. One is a round bottom vortex which is formed in a vessel as the fluid in a beaker is stirred with a glass rod. The surface of liquid in a beaker placed on a magnetic stirrer will be given a round bottom vortex by the revolution of a stirring element. When the rate of revolution is greater, then the vortex may become deeper proportionally. This round bottom vortex (hereafter it is called as "round vortex" for simplicity) is formed always in an unaffled vessel, so the liquid flow in the vessel might be a steady state. The axis of the revolution of the vortex is coincident with that of the stirring element of the mangetic stirrer.

Anothor type of vortex is a helical cone vortex which is appeared in a baffled vessel. This type of helical cone vortex (hereafter it is abbreviated to "helical vortex") is produced sometimes at about middle position between the baffle plates mounted on the side-wall of the vessel and the agitating shaft fixed with a paddle wheel or a turbine impeller. The paddle which has four plates being inclined at 45 degrees to the shaft and the impeller having six plates of curved blade were tested here. Each of them was immersed in about ten centimeters below the surface of fluid.

The informations of an agitated tank with the baffled and unbaffled are given in the famous book by Professor R.B. Bird[1] of Univ. of Wisconsin. However, in the descriptions of his book, the paddle and impeller are fixed on the shaft at near the bottom of a tank. The helical vortex could not be found in these circumstances. Fortunately, we met with the helical vortex at lower rate of revolution of the shaft mounted with the impeller about ten centimeters under the surface. The faster the rate of revolution of the shaft, the more the vortex becomes distinctive. According to the increasing of the rotating velocity, much more air is sucked in water. More than 300 rpm of the shaft speed the vessel is entirely

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covered with foam.

One of the purposes of this work is how to make a helical vortex in an agitating tank, and to outline the circumstances of the creation of the helical vortex. The detailed descriptions on the helical vortex shall be discussed in the next work.

Experimentals

The agitating tank is shown in Fig. 1. 50 l of plastic tank (A) was used in this experiment. It was fixed on a rack (B) made of steel, and four baffle plates were mounted on the inner side of the tank when they were needed. A suitable steel support (C) was placed on a rack to fix the driving mechanism. They are composed of a torque meter (E), a stepless speed change device (F) and a driving motor (G) which is mounted on the top of this equipment. A stable coupling (D) fitted with a long shaft for mounting a paddle or an impeller was placed between the support (C) and the torque meter (E).

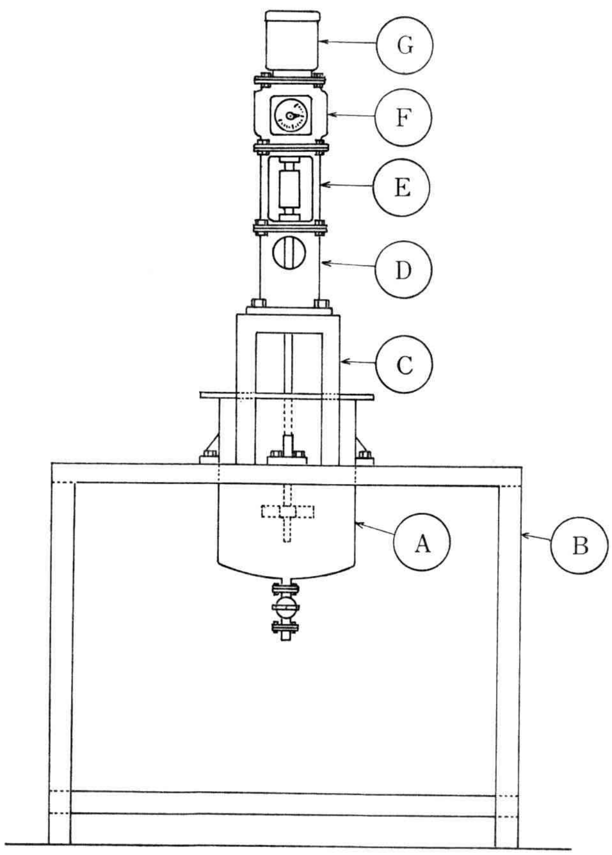


Fig. 1 An agitation tank assembly. (A) is the tank, (B) the steel rack, (C) the steel support, (D) the stable coupling, (E) the torque meter, (F) the stepless speed change device and (G) the driving motor.

Fig. 2 shows a 45° pitched paddle wheel at (a) and a curved blade impeller at (b) used here.

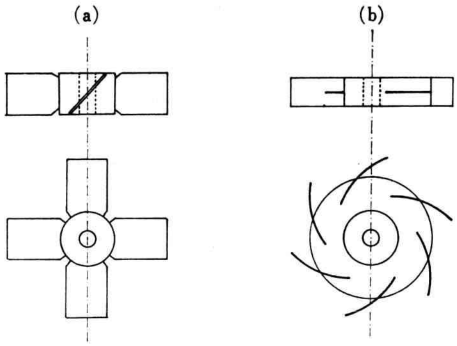


Fig. 2 The agitation elements used in this work. (a) is the paddle and (b) is the curved impeller.

The vortices resulted in this work are shown in Figs. 3 and 4.

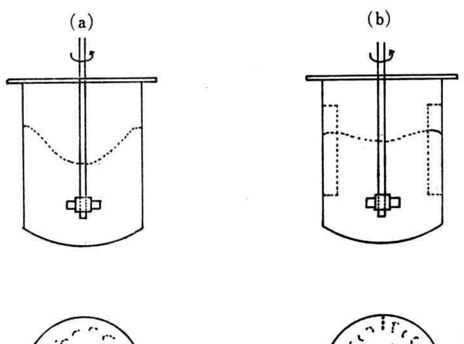


Fig. 3 The schematic diagram of the agitating tank with the paddle or impeller mounted near the bottom of the tank. (a) is the unbaffed tank and (b) is the baffled one.

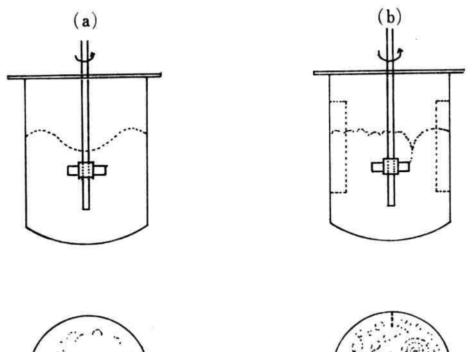
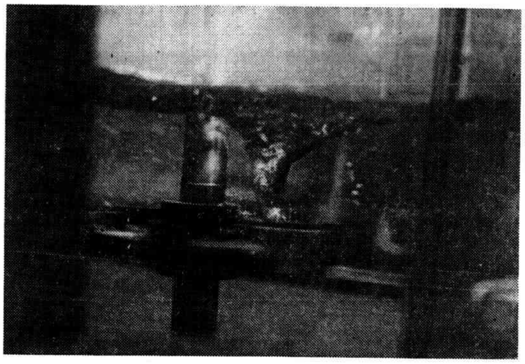


Fig. 4 The schematic diagram of the agitating tank with the paddle or impeller mounted at near the surface of fluid. (a) is the trivial vortex and (b) is the most interesting one taken up in this work.

Fig. 3 shows the trivial vortices which are given in most cases of the agitating tanks. In these, the paddle or impeller is located at near the bottom of the tank in each case of the baffled and the unbaffled one. While, Fig. 4 shows the cases of both the paddle and impeller are mounted on the shaft near the surface of the fluid. The vortex in Fig. 4 (a) is

(a)



(b)

Phot. 1 The photographs of the helical vortex. (a) shows a typical vortex and (b) is one which is just before broken down by the impeller.

the same shape as those in the Fig. 3. However, the vortex in Fig. 4 (b) is the most interesting one which is called as the "helical vortex" in this work. The photographs show the characteristic feature of the vortex. The circumstances in which the helical vortex would appear are so complicated that the detailed discussions on them may be postponed to the next work. Then, we may only report the creation of helical vortex in an agitating tank.

Discussions

The density of fluid may be assumed invariable in many cases. If no appreciable compression or expansion is shown in the liquid flow, we call it as "incompressible flow". Now it can be written the equation of a volume element in the fluid as

$$\rho \frac{d\boldsymbol{v}}{dt} = -\, grad\, p \tag{1}$$

and the using the Euler equation which is

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot grad)\, \boldsymbol{v} = -\frac{1}{\rho}\, grad\, p \tag{2}$$

The motion of fluid is in a gravitational field, so the acceleration of gravity $\boldsymbol{g}$, should be added the right hand of the above equation, then

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot grad)\, \boldsymbol{v} = -\frac{grad\, p}{\rho} + \boldsymbol{g} \tag{3}$$

Hence we can write down Bernoulli equation immediately by only replacing the heat function w by p/ρ

$$\frac{1}{2} v^2 + p/\rho + gz = \text{constant} \tag{4}$$

Now, we determine the shape of the surface of the fluid. At first, the fluid in a cylindrical vessel is rotating about the vertical axis z with a constant angluar velocity Ω. Then $v_x =$

$-y\Omega_1$, $v_y = x\,\Omega$ and $v_z = 0$. According to Bernoulli's equation, dp is of order of $\rho\, v^2$ in steady flow, so next identities are satisfied

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = x\,\Omega^2$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y} = y\,\Omega^2$$

and from Euler's equation (1), we get

$$\frac{1}{\rho}\frac{\partial p}{\partial z} + g = 0$$

The integral of these equations is

$$\frac{p}{\rho} = \frac{1}{2}\,\Omega^2 (x^2 + y^2) - gz + constant \tag{5}$$

At the free surface p=constant, so the surface is a paraboloid, and its origin is taken at the lowest point of the surface

$$z = \frac{1}{2}\frac{\Omega^2}{g}(x^2 + y^2) \tag{6}$$

In the second, the turbulent flow is rotational in nature. In general, the whole volume of the fluid may be divided into two different regions. One is the rotational flow region, while the other has no vorticity, which means curl $\boldsymbol{v}=0$ in the fluid. The non-zero vorticity of a finite domain may be created by the consequence of the formation of turbulent flow regarded as the motion of fluid satisfying Euler's equation. Then we may write concerning the velocity of circulation of the fluid as

$$\oint \boldsymbol{v}\cdot \boldsymbol{dl} = constant \tag{7}$$

where $\boldsymbol{dl}$ is an element of the length of the contour of vortex. One of the properties of the region of rotational turbulent flow is that the exchange of fluid between this domain and the surroundings may occur in one direction. If at any point on a streamline $w \neq 0$ would be granted, then the same is true at every point on this streamline. Conversely, if at any point on a streamline $w \neq 0$, then w does not vanish anywhere on that. This is the law of conservation of circulation. Such a distribution of w will be stable. Unfortunately, it has not yet provided us as affording a rigorous proof of the arguments.

To make clear the separation of vortex region, it is important to determine the position of this separating line on the surface of the fluid. The problem is reduced to seek the properties of the solutions of equations of motion in the boundary layer. The streamline begin to leave the surface layer and to enter the interior of the fluid in early stage. By the conservation of circulation, this penetration phenomenon may occur only by the direct mixing of fluid moving near the surface with the main stream. The equation in the boundary layer result that the tangential velocity component (v_x) in the boundary is compared with the component (v_y) normal to the surface of the body. It will be found that the relation between v_x and v_y exhibits the nature of the flow in the boundary layer, and that of all the points not lying in the immediate nighbourhood of singular points. But if $v_y \ll v_x$ it follows that the fluid moves along the surface, and moves slightly away from the surface, so that no separation takes place. The flow penetrates from the boundary layer towards the interior of the fluid. This is the case that the normal velocity component decreases to be small compared with the tagential component. The ratio v_y/v_x is of the order of $1/\sqrt{R}$, so that an increase of v_y means an increase by a factor of $\sqrt{R}$.

From the arguments above, there must be separation somewhere on the surface of the fluid. The stagnation point at which the fluid velocity is zero for potential flow. Consequently, for some value of x, the velocity U_x is zero at the outer of the boundary layer, the velocity in the nearest neighbour of the surface must be zero. This means mathematically that the derivative $\partial v_x/\partial y$ must vanish for some x which $U_x=0$. A consideration may be carried out in similar manner the derivative $\partial v_z/\partial y$ of the two velocity components v_x and v_z tangential to the surface. Then, rapid increase of pressue in the direction of its flow means a rapid decrease in velocity U. When a small distance $\Delta x = x_2 - x_1$, and the pressure p increase rapidly from p_1 to p_2 ($p_2 > p_1$), the fluid velocity outside the boundary layer falls from its initial value U_1 to a certainly smaller value U_2 over the same distance Δx. By Bernoull's equation,

$$\frac{1}{2}\,|U_1^2 - U_2^2| = \frac{p_2 - p_1}{\rho} \tag{8}$$

Since p is independent of y, the pressure gradient dp/dx in x direction is sufficiently high. To evaluate the differences in the velocity v in the boundary layer, we use the Bernoulli's relation as

$$\frac{1}{2}\,(v_2^2 - v_1^2) = -\frac{p_2 - p_1}{\rho} \tag{9}$$

Then, we get the next relation

$$v_2^2 = v_1^2 - (U_1^2 - U_2^2) \tag{10}$$

The velocity v_1 in the boundary layer is less than that of the main stream, and from the circumstances mentioned above, it may be

$$v_1^2 < (U_1^2 - U_2^2)$$

Then the velocity v_2 should be imaginary. The separation in the distance x should be resulted at the boundary. Another vortex[2)] can be found that as fluid is withdrawn from some liquid storage tank, and, as the liquid level drops, the vortex will ultimately reach the draw-off pipe. More details of the calculations will be postponed to a next paper.

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Nomenclatures

ρ: The mass per unit volume.
U: The velocity vector of the streamline.
p: The pressure.
$\mathbf{g}$: The gravitational vector.
Ω: The angular velocity of the strealmine.
$d\mathbf{l}$: The unit vector upon the tangential plane of circulation.
R: The Reynolds number.
U_x: The fluid velocity with x-component.
w: The heat function which is the sum of the internal energy per unit mass and p/ρ.

References

1) R.B. Bird, W.E. Steward and E.N. Lighfoot, "Transport Phenomena", John Wiley & Sons, Inc., N.Y., (1960).
2) I.J. Karassik, "Engineer's Guide to Centrifugal Pumps", McGraw-Hill, Inc., N.Y., (1964).