Behavior of Railway Cars in Running (Report 1)

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Abstract

The necessary conditions for railway cars are low cost and high performance. From the viewpoint of behavior of railway cars, derailment is the most important factor to be studied. In this report, the fundamental part of the study, concerning the phenomena between the wheel of railway cars and the rail, is explained.

Nomenclature

Notation	Unit	Definition
b	m	half distance between contact points of right and left wheel treads and rails in the lateral direction
F	kgf	force
f	kgf	creep coefficient
h	m	height concerning flange-rail contact
I_w	$\mathrm{kgf \cdot m \cdot s^2}$	moment of inertia of a wheelset around a vertical axis
k_y	kgf/m	lateral supporting stiffness of axle
k	$\mathrm{kgf \cdot m}$	supporting stiffness of axle in yaw
L	m	wave length
m_w	$\mathrm{kgf \cdot s^2/m}$	mass of a wheelset
P	kgf	wheel load
Q	kgf	side thrust of wheel
r	m	radius of wheel tread circle
t	s	time
v	m/s	forward speed of wheelset
X	m	longitudinal ordinate
x	m	longitudinal displacement
y	m	lateral displacement
y_w	m	lateral displacement of wheelset
z	m	vertical displacement
α	rad	flange angle
γ	rad	wheel tread inclination
δ	rad	angular constant
θ	rad	frictional angle; rotating angle of wheel
Ψ	rad	amplitude in yaw
ψ	rad	angular displacement in yaw; attack angle of a wheel to rail
ψ_0	rad	initial attack angle of a wheel to rail
ψ_w	rad	angular displacement of wheelset in yaw
ϕ	rad	angular displacement in roll
ω	rad/s	angular frequency

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1. Introduction

The necessary conditions for railway cars, like other transport media, are low cost with high performance. From the viewpoint of motion or vibration of railway cars, they are safety the first, passenger's comfort or cargo's undamage the second, etc. Safety means no derailment or no overturn, no crash, etc. To meet these conditions, velocity, acceleration or jerk of vibration of car body must be limited to a certain extent. So efforts are always made to transport speedily a large quantity, while keeping the abovementioned conditions.

This report, relating these conditions, discusses the behavior of railway cars in running, and especially attaching importance to derailment. It states the study of the characteristics of vibration and of the simulation technique which has been developed to analyze the motion of railway cars.

On thinking of the motion of railway cars, a vehicle body, for example, has six degrees of feedom; parallel motions in the lateral, vertical and longitudinal directions, and turning motions of yawing (around vertical axis), pitching (around lateral axis) and rolling (around longitudinal axis). Among them, the longitudinal motion is related to the force through couplers and to acceleration or deceleration of vehicle in running. Vertical and pitching motions in running of vehicle are mainly excited by vertical variation of track (called irregularity of longitudinal level). Lateral, yawing and rolling motions are produced with relation to lateral variation (irregularity of alignment), level variation between right and left rail (irregularity of cross level) and spacing variation between the two rails (irregularity of gauge).

On longitudinal motion(1), a vehicle motion in a train hauled by a locomotive, shock at the hitting with vehicle to vehicle at yard (place to assort freight cars), etc., and on vertical(2),(3), pitching motions, riding comfort (feeling of passengers against vibration), are main problematic points, respectively.

On the motion of a vehicle related with derailment, the difference of wheel loads of right wheel and left one and the side thrust of a wheel are considered to be main factors. So that the lateral, yawing and rolling motions, which affect on the above matters, are to be treated as simplified to three in this case, that is, lateral motion, rolling and yawing.

It is the basis of railways that steel wheels, fixed by an axle, roll on rails or track, and suspension gears, which are equipped on the bearings of wheelsets (a wheelset means a set of two wheels and an axle), support a bogie frame or a vehicle body. This running wheelset has a tendency to produce a self-exited motion called hunting of a wheelset.

2. Motion of a wheelset in rotating

2.1 Orientation for a wheelset

A wheelset is constantly guided to run along the center line of track by the effect of its wheel tread gradient in section. Figure 2.1 shows a dynamic model of a wheelset. Coordinates are chosen to right-hand-system and are oriented as follows; advancing direction is positive direction of x-axis, left hand direction for advancing is positive direction of y-axis and upward direction is z-axis. Positive direction for rotation in yawing (ψ), is from x-axis to y-axis in rolling (ϕ) is from y-axis to z-axis and that in rotation around y-axis is from z-axis to x-axis. The origin of co-ordinates is put at the center of gravity of a wheelset at its neutral position and the axises of co-ordinates hold the absolute directions.

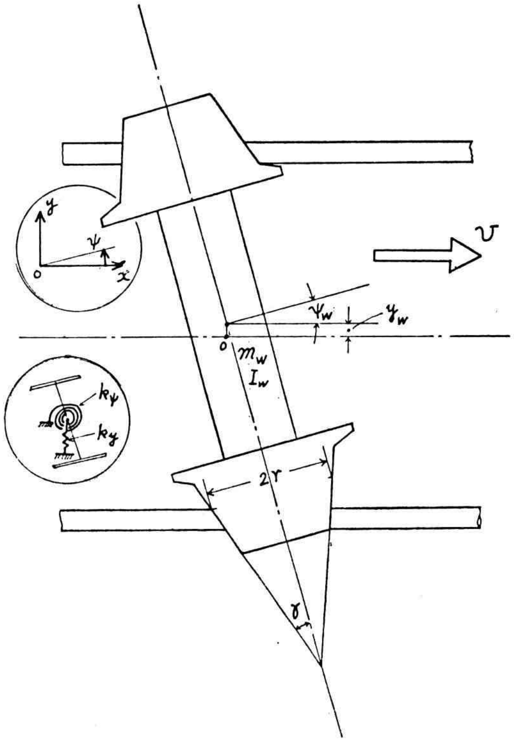


Fig. 2.1 Dynamic model of a wheelset

2.2 Creep coefficient between wheel and rail

When an advancing wheelset is pushed in lateral by a constant force, it is experimentally and theoretically recognized that the wheelset moves laterally with a speed related to the strength of the applied force and the advancing speed of the wheelset. In small range of displacement, this lateral speed of the wheelset is assumed to be proportional to the force strength and the advancing speed of the wheelset, and the following formula holds,

$$F=f\frac{\dot{y}_w}{v} \tag{2.1}$$

where F stands for the force applied in lateral and f is a proportional constant (y_w and v are in Fig. 2.1). f is called creep coefficient of a wheelset to rails(4) and its unit is the same as that of force. This relation is also supposed to hold in the longitudinal direction.

2.3 Hunting motion of a wheelset(4)

In kinematics, an advancing wheelset, once deviating from its neutral position, runs in sinusoidal way like snake movement. In dynamics, the inertia force of the wheelset in lateral direction acts to increase the sinusoidal motion amplitude, because of existing creep. This means a running wheelset has a tendency to produce a self-exited motion which is called the hunting motion of a wheelset.

Relative speed of rail to wheel is expressed as $(\dot{y}_w\text{-}v\psi_w)$ in the lateral direction, and so the following expression (2.2) will be obtained by the balancing of inertia force, creep force and spring force. For yawing motion of a wheelset, considering one wheel of a wheelset and rail in the longitudinal direction, the relative speed will be expressed as

$$b\dot{\psi}_w-\{r\omega-(r+\gamma\, y_w)\omega\} \rightarrow b\dot{\psi}_w+\gamma\omega\, y_w$$

From the balancing of moments in yaw, and by the relation $r\omega=v$, (2.3) is obtained.

$$m_w\ddot{y}_w+f\left(\frac{\dot{y}_w}{v}-\psi_w\right)+k_y y_w=0 \tag{2.2}$$

$$I_w\ddot{\psi}_w+fb\left(\frac{b\dot{\psi}_w}{v}+\frac{\gamma}{r}y_w\right)+k_\psi\psi_w=0 \tag{2.3}$$

Putting $y_w=Ye^{pt}$, $\psi_w=\Psi e^{pt}$, the characteristic equation will be

$$\begin{vmatrix} m_w p^2+\dfrac{f}{v}p+k_y & -f \\ fb\dfrac{\gamma}{r} & I_w p^2+f\dfrac{b^2}{v}p+k_\psi \end{vmatrix}=0 \tag{2.4}$$

$$\left(m_w p^2+\frac{f}{v}p+k_y\right)\left(I_w p^2+\frac{fb^2}{v}p+k_\psi\right)+\frac{f^2 b\gamma}{r}=0 \tag{2.5}$$

Letting $k_y=0$ and $k_\psi=0$ in this equation, it shows that the vibration is not convergent because of lacking of first order term of p. The spring forces to the wheelset, if they act suitablly, may suppress the amplitude of the hunting, that means they make the wheelset motion stable.

Neglecting inertia forces and spring forces at the equations (2.2), (2.3), and substituting that $v=dx/dt$, we get the following equations.

$$\frac{dy_w}{dx}-\psi_w=0 \tag{2.6}$$

$$b\frac{d\psi_w}{dx}+\frac{\gamma}{r}y_w=0 \tag{2.7}$$

Eliminating ψ_w from these equations;

$$\frac{d^2y_w}{dx^2}+\frac{\gamma}{br}y_w=0\,. \tag{2.8}$$

Solving this equation, we get,

$$y_w = y_0 \sin\left(\frac{2\pi}{L} \cdot x + \delta\right) \tag{2.9}$$

where

$$L = 2\pi\sqrt{\frac{br}{\gamma}}. \tag{2.10}$$

This shows that a wheelset motion in running is nearly sinusoidal one with constant wave length, which means its frequency changes in proportion to its running speed.

2.4 Condition of track

Same as auto road, railway track maintenance is constantly carried out, and so called track irregularity has been kept under certain limits. It is said that the relation between wave length and amplitude of track irregularity is almost in proportion as shown in Fig. 2.2.⁽⁵⁾

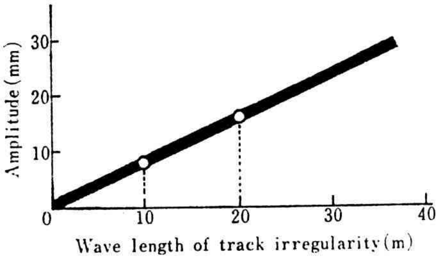


Fig. 2.2 Relation between wave length and amplitude of track irregularity

On considering a certain frequency, such as natural frequency of car body, the amplitude of forced vibration by track irregularity increases as speed up of the railway car.

Curved track has cant which balances centrifugal force of a car with a scheduled speed, and has also slack which means track gauge is slightly widened by certain amount. Derailment often takes place at curved track. Some of the cases are caused by wheel load off due to unadequate, too high or too low to planned cant, speed of railway cars passing through a curved track. Slack makes it wide for wheelset to move in the lateral direction, which means the amplitude of wheelset lateral vibration, when occurred, grows larger.

2.5 Derailment of a wheel⁽⁵⁾

Derailment is utlimately the interaction between wheel and rail. Figure 2.3 shows this relation on two dimensional plane. Let Q side thrust of wheel to rail and P wheel load to rail, in case that $Q/P<\tan(\alpha-\theta)$ which means the wheel slides downwards, no derailment coccurs, in other case that $Q/P>\tan(\alpha+\theta)$ which means the wheel slides upwards, derailment does occur, and in case

$$\tan(\alpha-\theta) \leqq Q/P \leqq \tan(\alpha+\theta) \tag{2.11}$$

no slide occurs between wheel and rail, where θ is friction angle for μ, which is an apparent frictional coefficient between wheel and rail as shown at point A in Fig. 2.3.

Figure 2.4 shows the two limits of Q/p in (2.11), μ as a parameter, where broken lines indicate the upper limit in the expression (2.11).

In range shown by the formula (2.11), derailment occurs at certain conditions. It is stated in detail in the subsequent sections.

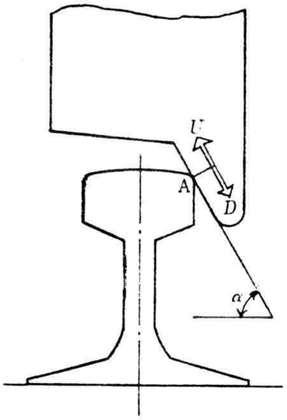


Fig. 2.3 Contact of wheel flange and rail on almost derailment

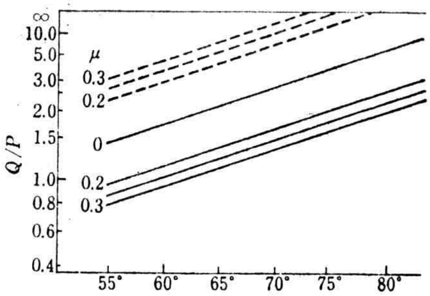


Fig. 2.4 Relation between wheel flange angle and derailment coefficient with frictional coefficient as a parameter

2.6 Rail attack angle of a wheel⁽⁵⁾

Angle ψ_0 in Fig. 2.5 is called an attack angle of a wheel to rail. In this situation, the contact point between wheel flange and rail comes little forwards from the center of wheel. Letting the X-ordinate of the contact point l,

$$l \doteqdot (r \tan\alpha)\cdot\psi_0 \tag{2.12}$$

Letting the height of the contact point as h in the wheel radius direction, where $h=0$ at $\psi_0=0$.

$$h \doteqdot \frac{1}{2}(r \tan^2\alpha)\cdot\psi_0{}^2 \tag{2.13}$$

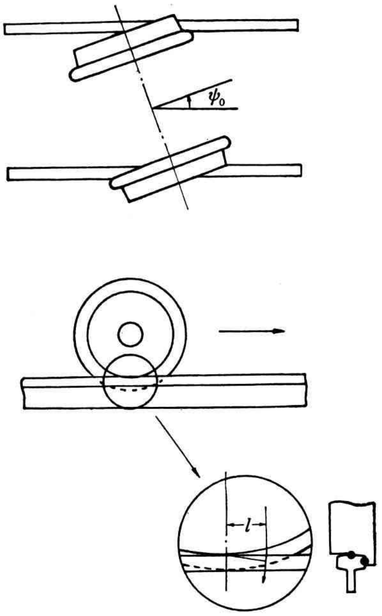
Assuming $h=0.02$ as limit, then

$$\psi_{cr}=\frac{0.2}{\sqrt{r}\cdot\tan\alpha} \tag{2.14}$$

Figure 2.6 shows this limit line, when $r=0.43$.

2.7 Motion of a wheelset with no slide

In the range shown by (2.11), a wheelset which contacts rail with an attack angle ψ_0, will trace a certain line. Assuming that a wheelset is advancing as in Fig. 2.5, the left wheel will



←Fig. 2.5 Three dimensional relation of wheel and rail

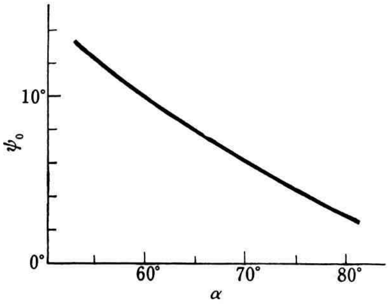


Fig. 2.6 Relation of wheel flange angle and maximum attack angle at $h=0.02$ [m] from the viewpoint of flange-rail contact

begin to climb rail. At some position of the wheelset, the left wheel advances $(r+z)d\theta$, and the right wheel does $rd\theta$, where z is the displacement of the left wheel in upward direction and $d\theta$ is rotating infinitesimal angle of the wheelset. Then the following relations hold.

$$-d\psi = z\,d\theta/(2b) \tag{2.15}$$

$$dx = r\,d\theta \tag{2.16}$$

$$\frac{dy}{dx} = \psi \tag{2.17}$$

$$z = y \tan\alpha \tag{2.18}$$

Eliminating z, ψ, x and $d\theta$ from these equations, we get:

$$\frac{d^2y}{dx^2} + \frac{\tan\alpha}{2br}y = 0 \tag{2.19}$$

Solving this equation, we get;

$$y = (\psi_0/\omega)\sin\omega x \tag{2.20}$$

where,

$$\omega = \sqrt{\frac{\tan\alpha}{2br}} \tag{2.21}$$

And then,

$$z = h\sin\omega x \tag{2.22}$$

where,

$$h = \psi_0\sqrt{2br\tan\alpha} \tag{2.23}$$

Figure 2.7 shows the traces of left wheel in x-z and x-y planes, and Fig. 2.8 shows an example of limit that the left wheel does not derail.

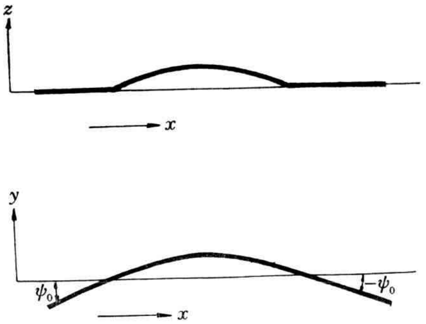


Fig. 2.7 Motion of left wheel with no slide at wheel-rail contact

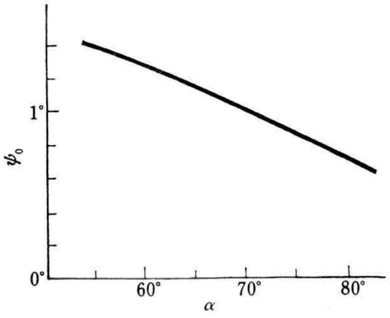


Fig. 2.8 Relation of wheel flange angle and maximum attack angle ($z_{\max}=0.02$ in Fig. 2.7)

3. Remarks

The relation between wheel and rail is essential to derailment of railway cars. It has been studied from many points of view and some of the results are shown on this report.

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