

On the Contactless Measurement of Electric Conductivity of Semiconductor Circular Wafer

I. Theoretical Consideration

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Abstract

Authors derived an electromagnetic theory of the contactless measuring method of electric conductivity of semiconductor circular wafer referring to the arrangement shown in Fig. 1, and have got the result in the case $d'=d$:

$$V = -6.120 \times 10^{-12} f^2 \sigma a^2 n a'^2 n' I t \frac{b^4}{d^2(d^2+b^2)^2} \quad [V] \quad [16]$$

where V is electromotive force induced along measuring coil due to the eddy current in the wafer induced by exciting coil, for the lower frequency range:

$$f \ll 50\text{MHz}, \quad \lambda \gg 6\text{m}, \quad \text{for } r \approx 1\text{m}, \quad [1]$$

where r is a symbolic representation of scale of arbitrary element used in this arrangement.

We discussed practical measurement of V , especially separation of V and V_0 which is electromotive force induced by direct field from exciting coil to measuring coil, using the phase difference between V and V_0 . We use SI unit system and time factor $e^{j\omega t}$ in this paper.

1. Introduction

In the usual contact method measuring electric conductivity σ [S/m] of semiconductor wafer, it would be unavoidable to pollute and damage the wafer more or less by contact of electrodes on it, and then, some errors would be introduced in measured values.

Therefore it is very important to establish a contactless measuring method of σ of the semiconductor wafer theoretically and practically¹⁾.

Authors will give here an electromagnetic theory of measurement of σ of the semiconductor circular wafer measuring eddy current induced in it by exciting coil with measuring coil. We neglect here the radiation term compared with induction term, therefore all sizes represented symbolically with r (which may represent any one of $a, a', l, l', d, d', \sqrt{(2\pi a n)^2 + 1^2}, \sqrt{(2\pi a' n')^2 + 1'^2}, t$ in Fig. 1.) excepting the radius of wafer b which may be large indefinitely, should be much smaller than the wave length $\lambda=c/f$, where $c=2.99792 \times 10^8$ m/s is the light velocity in the vacuum, f the frequency in Hz:

$$kr = \frac{2\pi}{\lambda} r \ll 1 \quad \text{then} \quad r \ll \frac{\lambda}{2\pi} \doteq \frac{50 \times 10^6}{f}. \quad [m] \quad [1]$$

If we assume r is about 1 m, $f \ll 50\text{MHz}$ or $\lambda \gg 6\text{m}$. Of cause, we may extend the usable frequency range minimizing the sizes of element in the measuring device referring to [1].

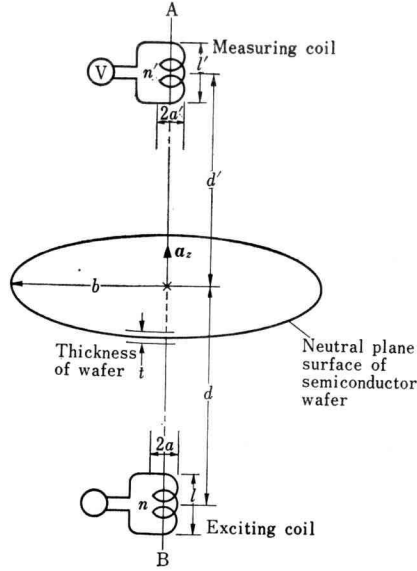


Fig. 1 Schematic diagram of contactless measurement method of σ

2. Measuring method and its theory

Fig. 1 shows the schematic display of contactless measuring method of electric conductivity of semiconductor circular wafer, in which an exciting coil, semiconductor circular wafer and measuring coil are given in their own position. The central lines of the exciting coil, measuring coil and the semiconductor circular wafer are lined up on a straight line along z axis. a, l, n are radius, length, number of turns of the exciting coil respectively; a', l', n' are the same of the measuring coil. a, l, a', l' and thickness t of wafer must be sufficiently small compared with d and d' which are distances of two coils to the neutral plane surface of the semiconductor circular wafer, because we assume here the coils are magnetic dipoles with the magnetic dipole moments²⁾:

$$\mathbf{p}_m = \mathbf{a}_z p_m = \mathbf{a}_z \mu_0 \pi a^2 n I \text{ [Wb m]}, \quad \mathbf{p}_m' = \mathbf{a}_z p_m' = \mathbf{a}_z \mu_0 \pi a'^2 n' I' \text{ [Wb m]} \quad [2]$$

respectively. We assume also that wafer is not magnetic, i.e., its permeability is μ_0 .

Exciting coil with magnetic dipole moment $\mathbf{p}_m = \mathbf{a}_z p_m$ will make the magnetic field $\mathbf{H}(Q)$ in any point Q on the neutral plane surface of semiconductor circular wafer³⁾ (Fig. 2):

$$\mathbf{H}(Q) = \mathbf{a}_r \frac{p_m \cos \theta}{2\pi \mu_0 r^3} + \mathbf{a}_\theta \frac{p_m \sin \theta}{4\pi \mu_0 r^3}, \quad [\text{A/m}] \quad [3]$$

and then, magnetic flux density at that point would be

$$\mathbf{B}(Q) = \mu_0 \mathbf{H}(Q) = \mathbf{a}_r \frac{p_m \cos \theta}{2\pi r^3} + \mathbf{a}_\theta \frac{p_m \sin \theta}{4\pi r^3}. \quad [\text{Wb/m}^2] \quad [4]$$

Normal component of magnetic flux density pass upward through at the point Q would be

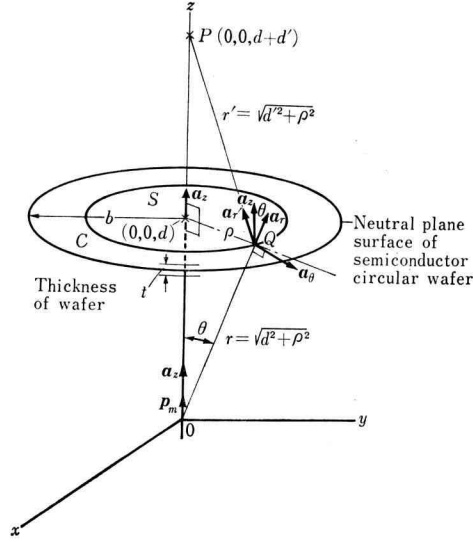


Fig. 2 For the theoretical consideration

$$B_z(Q) = \mathbf{a}_z \cdot \mathbf{B}(Q) = \mathbf{a}_z \cdot \mathbf{a}_r \frac{\dot{p}_m d}{2\pi r^4} + \mathbf{a}_z \cdot \mathbf{a}_\theta \frac{\dot{p}_m \rho}{4\pi r^4} = \frac{3\dot{p}_m d^2}{4\pi r^5} - \frac{\dot{p}_m}{4\pi r^3}. \quad [\text{Wb/m}^2] \quad [5]$$

Total magnetic flux pass upward through a circle of radius ρ on the neutral plane surface of the semiconductor circular wafer would be

$$\Phi_m(\rho) = \int_0^\rho B_z(Q) 2\pi \rho d\rho = \frac{6\pi \dot{p}_m d^2}{4\pi} \int_0^\rho \frac{\rho d\rho}{r^5} - \frac{2\pi \dot{p}_m}{4\pi} \int_0^\rho \frac{\rho d\rho}{r^3}, \quad [\text{Wb}] \quad [6]$$

If we put $d^2 + \rho^2 = r^2$; then, $\rho d\rho = r dr$; $\rho=0$, $r=d$; $\rho=\rho$, $r=\sqrt{d^2 + \rho^2}$;

$$\Phi_m(\rho) = \frac{6\pi \dot{p}_m d^2}{4\pi} \int_d^{\sqrt{d^2 + \rho^2}} \frac{r dr}{r^5} - \frac{2\pi \dot{p}_m}{4\pi} \int_d^{\sqrt{d^2 + \rho^2}} \frac{r dr}{r^3} = \frac{\dot{p}_m \rho^2}{2\sqrt{d^2 + \rho^2}} \mathbf{z}. \quad [\text{Wb}] \quad [7]$$

According to Faraday's law of induction, electromotive force $V(\rho)$ induced along a coaxial circle C of radius ρ on the neutral plane surface of the semiconductor circular wafer would be:

$$V(\rho) = -\frac{d\Phi_m}{dt} = -j\omega \Phi_m(\rho) = -j\omega \frac{\dot{p}_m \rho^2}{2\sqrt{d^2 + \rho^2}} \mathbf{z}. \quad [\text{V}] \quad [8]$$

Therefore electric field $\mathbf{E}(\rho)$ along C has to be (see Appendix I);

$$\mathbf{E}(\rho) = \mathbf{a}_\varphi \frac{V(\rho)}{2\pi \rho} = -\mathbf{a}_\varphi j\omega \frac{\dot{p}_m \rho}{4\pi \sqrt{d^2 + \rho^2}} \mathbf{z} = -\mathbf{a}_\varphi j\omega \frac{\dot{p}_m \sin \theta}{4\pi r^2}. \quad [\text{V/m}] \quad [9]$$

The electric current density $\mathbf{J}(\rho)$ in the semiconductor circular wafer along C would be:

$$\mathbf{J}(\rho) = \sigma \mathbf{E}(\rho) = -\mathbf{a}_\varphi j\omega \frac{\sigma \dot{\phi}_m \rho}{4\pi \sqrt{d^2 + \rho^2}} \mathbf{z}, \quad [\text{A/m}^2] \quad [10]$$

And, current surface density $\mathbf{K}(\rho)$ in the semiconductor circular wafer would be:

$$\mathbf{K}(\rho) = \mathbf{J}(\rho)t = -\mathbf{a}_\varphi j\omega \frac{\sigma \dot{\phi}_m \rho t}{4\pi \sqrt{d^2 + \rho^2}} \mathbf{z}, \quad [\text{A/m}] \quad [11]$$

According to Biot-Savart's law, magnetic field $\mathbf{H}(P)$ produced on P which is the center of measuring coil would be:

$$\mathbf{H}(P) = \frac{1}{4\pi} \int_s \frac{\mathbf{K}(\rho) \times \mathbf{a}_{r'}}{r'^2} dS = \mathbf{a}_z \frac{-j\omega \sigma \dot{\phi}_m t}{8\pi} \int_0^b \frac{\rho^3 d\rho}{\sqrt{d^2 + \rho^2}^3 \sqrt{d'^2 + \rho^2}} \mathbf{z}. \quad [\text{A/m}] \quad [12]$$

Because, \mathbf{a}_ρ component would be zero when it is integrated with respect to φ due to its symmetry.

Now, we will put $d'=d$ for the sake of simplicity of calculation (see Appendix II),

$$\mathbf{H}(P) = \mathbf{a}_z \frac{-j\omega \sigma \dot{\phi}_m t}{8\pi} \int_0^b \frac{\rho^3 d\rho}{(d^2 + \rho^2)^3}. \quad [\text{A/m}] \quad [13]$$

According to the similar assumption in calculation of [6], it may be calculated as

$$\mathbf{H}(P) = \mathbf{a}_z H(P) = \mathbf{a}_z \frac{-j\omega \sigma \dot{\phi}_m t}{32\pi} \frac{b^4}{d^2(d^2 + b^2)^2}. \quad [\text{A/m}] \quad [14]$$

Therefore, magnetic flux density at P would be

$$\mathbf{B}(P) = \mathbf{a}_z \frac{-j\omega \mu_0 \sigma \dot{\phi}_m t}{32\pi} \frac{b^4}{d^2(d^2 + b^2)^2}. \quad [\text{Wb/m}^2] \quad [15]$$

Then the electromotive force $V[\text{V}]$ induced along the measuring coil due to Faraday's law of induction:

$$\begin{aligned} V &= -j\omega \pi a'^2 n' B(P) \\ &= -3.876 f^2 \mu_0^2 \sigma a^2 n a'^2 n' It \frac{b^4}{d^2(d^2 + b^2)^2} \\ &= -6.120 \times 10^{-12} f^2 \sigma a^2 n a'^2 n' It \frac{b^4}{d^2(d^2 + b^2)^2}. \quad [\text{V}] \end{aligned} \quad [16]$$

3. Considerations on practical measurement

3.1 Quantitative consideration

In practical measurement of V , measuring coil accepts not only $\mathbf{H}(P)$, but also direct field $\mathbf{H}_0(P)$ from exciting coil to measuring coil (for $d'=d$):

$$\mathbf{H}_0(P) = \mathbf{a}_z H_0(P) = \mathbf{a}_z \frac{\dot{\phi}_m}{2\pi \mu_0 (2d)^3} = \mathbf{a}_z \frac{\dot{\phi}_m}{16\pi \mu_0 d^3}. \quad [\text{A/m}] \quad [17]$$

And electromotive force induced in the measuring coil due to $H_0(P)$ would be

$$\begin{aligned} V_0 &= -j\omega\pi a'^2 n' \mu_0 H_0(P) = \frac{-j\omega\pi a'^2 n' \mu_0 \pi a^2 n I}{16\pi d^3} \\ &= -\frac{j\omega\pi \mu_0 a^2 n a'^2 n' I}{16d^3}. \quad [V] \end{aligned} \quad [18]$$

We will here compare V and V_0 or $H(P)$ and $H_0(P)$ in the special case of $b=\infty$:

$$\frac{V}{V_0} = \frac{H(P)}{H_0(P)} = -j\omega 6.28 \times 10^{-7} d \sigma t. \quad [19]$$

We may find the fact that V or $H(P)$ lead in the phase just 90° than V_0 or $H_0(P)$ respectively. Fig. 3 shows the vector diagram of I , V and V_0 . Fig. 4 shows the relation of $|V/V_0|$ or $|H(P)/H_0(P)|$ and σ , in a special case of $f=50\text{MHz}$, \dots , $t=1\text{ mm}$, $d=1\text{ cm}$. In practical measurement, when σ is comparatively large rather large values of f , t , or d should be selected; on the contrary, when σ is comparatively small, rather small values of f , t or d should be selected. It will be convenient to change number of thin wafers for the variation of t .

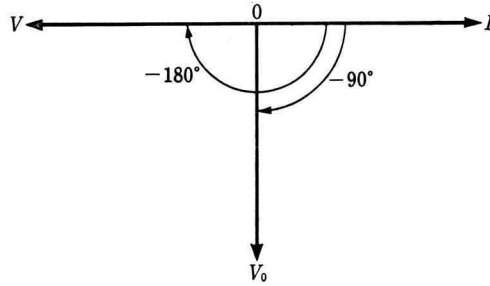


Fig. 3 Vector diagram of I , V , V_0

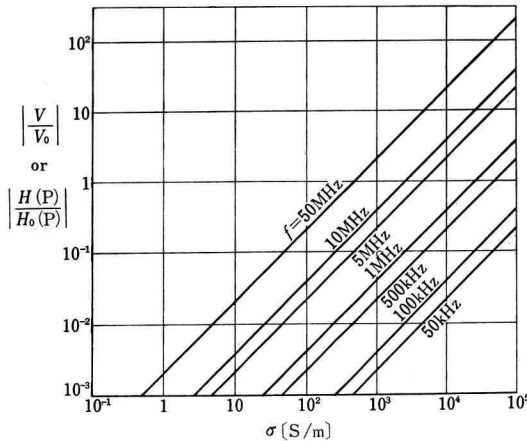


Fig. 4 Relation between $|V/V_0|$ (or $|H(P)/H_0(P)|$) and σ for $t=1\text{ mm}$, $d=1\text{ cm}$ and various value of f

3.2 Separation of V and V_0 using their phase difference

Let us input the voltage proportional to the current I of exciting coil on the horizontal axis of rectangular cathode ray tube oscillograph and receiving voltage of measuring coil on the vertical axis. Then the Lissage figure would be like as shown in Fig. 5. V_0 is

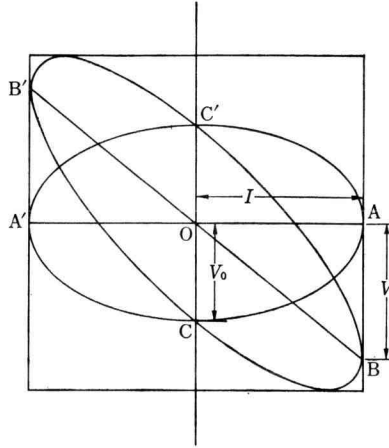


Fig. 5 Lissage figure, I is current in the exciting coil, V_0 the induced voltage in the measuring coil by direct magnetic field from exciting coil, V the voltage in the measuring coil due to eddy current in the wafer induced by exciting coil.

proportional to $-jI$, then V_0 - I figure would be ellipse or circle $ACA'C'$, and V is proportional to $-I$, then V - I figure would be a straight line BOB' , with negative inclination. Actual figure would be inclined ellipse $BCB'C'$. Therefore we may estimate V by AB , V_0 by OC .

Appendix I Verification of correctness of [9] from [4]

Explanation from [4] through [9] is very practical, but it is difficult to say correct to derive [9] from [8]. Therefore we will here derive [9] from [4] directly. The 4th equation⁴⁾ of Maxwell's equations which is originally a differential form of Faraday's law of induction:

$$\text{rot } \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t} \quad [\text{I-1}]$$

may be written in the spherical coordinates system (r, θ, φ) as

$$\begin{aligned} & \mathbf{a}_r \frac{1}{r \sin \theta} \left\{ \frac{\partial(\sin \theta E_\varphi)}{\partial \theta} - \frac{\partial E_\theta}{\partial \varphi} \right\} + \mathbf{a}_\theta \frac{1}{r \sin \theta} \left\{ \frac{\partial E_r}{\partial \varphi} - \sin \theta \frac{\partial(r E_\varphi)}{\partial r} \right\} \\ & + \mathbf{a}_\varphi \frac{1}{r} \left\{ \frac{\partial(r E_\theta)}{\partial r} - \frac{\partial E_r}{\partial \theta} \right\} \\ & = -\mathbf{a}_r j \omega \frac{\dot{\phi}_m \cos \theta}{2\pi r^3} - \mathbf{a}_\theta j \omega \frac{\dot{\phi}_m \sin \theta}{4\pi r^3}, \end{aligned}$$

in which [4] is introduced in right side. The 2nd term of \mathbf{a}_r component and 1st term of \mathbf{a}_θ component have to be zero, because of the symmetry of the phenomena with respect to φ . Comparing both sides, we may get the following 3 relations:

$$\frac{1}{r \sin \theta} \frac{\partial(\sin \theta E_\varphi)}{\partial \theta} = -j\omega \frac{p_m \cos \theta}{2\pi r^3}, \quad [\text{I-2}]$$

$$\frac{1}{r} \frac{\partial(r E_\varphi)}{\partial r} = j\omega \frac{p_m \sin \theta}{4\pi r^3}, \quad [\text{I-3}]$$

$$\frac{\partial(r E_\theta)}{\partial r} - \frac{\partial E_r}{\partial \theta} = 0. \quad [\text{I-4}]$$

Integrating [I-2] with respect to θ ,

$$\begin{aligned} \sin \theta E_\varphi &= -j\omega \frac{p_m}{2\pi r^2} \int \sin \theta \cos \theta d\theta = -j\omega \frac{p_m}{2\pi r^2} \frac{\sin^2 \theta}{2}, \\ \therefore E_\varphi &= -j\omega \frac{p_m \sin \theta}{4\pi r^2}. \end{aligned} \quad [\text{I-5}]$$

Next, integrating [I-3] with respect to r ,

$$\begin{aligned} r E_\varphi &= j\omega \frac{p_m \sin \theta}{4\pi} \int \frac{dr}{r^2} = j\omega \frac{p_m \sin \theta}{4\pi} \frac{-1}{r} = -j\omega \frac{p_m \sin \theta}{4\pi r}, \\ \therefore E_\varphi &= -j\omega \frac{p_m \sin \theta}{4\pi r^2}, \end{aligned}$$

which is the same with [I-5] at all. Finally, [I-4] satisfied by

$$E_r = 0, \quad E_\theta = 0, \quad [\text{I-6}]$$

Therefore we may get

$$\mathbf{E} = \mathbf{a}_\varphi E = -\mathbf{a}_\varphi j\omega \frac{p_m \sin \theta}{4\pi r^2}, \quad [9]$$

Thus, we may verify the correctness of [9] in such a way.

Appendix II

In this article, we assumed $d' = d$ for the sake of simplicity of calculation. But we may extend this condition to

$$d'^2 - d^2 \ll d^2 \text{ or } d' - d \ll d/2$$

as follows:

$$\int_0^b \frac{\rho^3 d\rho}{\sqrt{d^2 + \rho^2}^3 \sqrt{d'^2 + \rho^2}^3} = \int_0^b \frac{\rho^3 d\rho}{\sqrt{d^2 + \rho^2}^6 \sqrt{1 + \frac{d'^2 - d^2}{d^2 + \rho^2}}}. \quad [\text{II-1}]$$

According to binomial theorem⁵⁾:

$$\left(1 + \frac{d'^2 - d^2}{d^2 + \rho^2}\right)^{-3/2} = 1 - \frac{3}{2} \frac{d'^2 - d^2}{d^2 + \rho^2} + \dots \quad : |d' - d| \ll \frac{d}{2}, \quad [\text{II-2}]$$

Introducing [II-2] into [II-1] and calculating,

$$= \frac{b^4}{4d^2(d^2 + b^2)^2} - \frac{d'^2 - d^2}{8} \left(\frac{1}{d^4} - \frac{d^2 + 3b^2}{(d^2 + b^2)^3} \right)$$

is obtained.

Thus we may have the following extended formulae:

$$H(P) = \alpha_s \frac{-j\omega\sigma p_m t}{32\pi} \left\{ \frac{b^4}{d^2(d^2 + b^2)^2} - \frac{d'^2 - d^2}{2} \left(\frac{1}{d^4} - \frac{d^2 + 3b^2}{(d^2 + b^2)^3} \right) \right\}, \quad [14']$$

$$V = -6.120 \times 10^{-12} f^2 \sigma a^2 n a' n' I t \left\{ \frac{b^4}{d^2(d^2 + b^2)^2} - \frac{d'^2 - d^2}{2} \left(\frac{1}{d^4} - \frac{d^2 + 3b^2}{(d^2 + b^2)^3} \right) \right\} \quad [16']$$

for $d' - d \ll d/2$.

Appendix III

Relation between b and d obtaining maximum of V

$$0 = \frac{\partial \Phi_m(\rho)}{\partial \rho} = p_m \frac{2\rho \sqrt{d^2 + \rho^2}^3 - 3\rho^3 \sqrt{d^2 + \rho^2}}{2(d^2 + \rho^2)^3} = p_m \frac{\rho \sqrt{d^2 + \rho^2} (2d^2 - \rho^2)}{2(d^2 + \rho^2)^3}$$

$$\therefore \rho = b = \sqrt{2d}$$

If we use such a condition, V would be maximum, then measurement would be easy.

References

- 1) Horiguchi, F., Furukawa, S. and Sugii, T.: "Contactless measurement of sheet conductivity and mobility of semiconductor wafer by using eddy current", Trans. IECE Japan, '80/2 Vol. J63-c No. 2 p. 73 (1980).
- 2) Murakami, I.: "Theory of electricity and magnetism", Maruzen, p. 155 (1978).
- 3) ditto, p. 124.
- 4) ditto, pp. 183 and 203.
- 5) Binomial theorem:

$$(a+b)^n = a^n + {}_nC_1 a^{n-1}b + {}_nC_2 a^{n-2}b^2 + \dots + {}_nC_{n-1} a b^{n-1} + b^n$$

where

$${}_nC_i = \frac{n!}{(n-i)!i!} = \frac{n(n-1)\dots(n-i+1)}{i!}.$$