

# Proposition of Power Coaxial Cable Feeding Method for A.C. Electric Railroad

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## Abstract

This paper describes the new a.c. feeding method using power coaxial cable (CC), in which the inner and outer conductors of CC are connected to trolley wire and rail, respectively, by the feeding branch line established at suitable intervals.

The author has calculated the CC feeding circuits and obtained the characteristics for the power feeding and the electromagnetic induction. The results for this system are shown compared with the results for a feeding system using booster transformer and auto-transformer, respectively. The results for the CC feeding system show excellent characteristics. It is also clarified that shortening the interval between feeding branch lines results in better characteristics.

## Introduction

The single phase a.c. electrification of an electric railroad has the advantage that it is economical and suitable for high-speed and large capacity railroad. However, it has the disadvantage that an electromagnetic interference arises in neighbouring induced circuits. Therefore, various kinds of equipment in order to reduce inductive interference are inserted in the feeding circuits. The feeding systems in a.c. railroad which have already been put into practice in Japan are the booster transformer (BT) feeding system used in the New Tokaido Trunk Line and the unity turn-ratio auto-transformer (AT1) feeding system used in the New Sanyo Trunk Line and the New Tohoku Trunk Line.

This paper describes the new a.c. feeding method using power coaxial cable (CC), as shown in Fig. 1. In this method, the inner and outer conductors of CC are connected to trolley wire and rail, respectively, by the feeding branch line (FBL) established at suitable intervals.

This system may result in the excellent characteristics for both the power feeding and the electromagnetic induction. Therefore, the author has calculated the CC feeding circuits and obtained the characteristics. Furthermore, the results for this system are compared with the results for a feeding system using BT, AT1 and the auto-transformer of turn-ratio  $a=1.5, 2.0$  or  $3.0$  (AT $a$ ), respectively. From those results, it is clarified that the CC feeding method is very useful for a.c. electric railroad.

## Method of Analysis

The analysis of the a.c. feeding circuits (for example, as shown in Fig. 1) is greatly complicated by the following factors.

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- (1) The feeding circuits are in a multi-conductor system which consists of trolley wire, rail and so on.
- (2) Various devices treated as a lumped-constant circuit, (for example, FBL), are included in the midst of the lines.
- (3) The leakage conductance for earth of rail is large.
- (4) The load (electric railcar) must be supplied with a constant power so that it may move at a constant speed.

Thus, as a method for analyzing the feeding circuits, the author has developed "the method of applying the four-terminal constants theory to a multi-conductor system."<sup>(1)(2)</sup> In this method, the total length of the multi-conductor lines is divided into equal sections and each section is represented by four-terminal matrices. A system is then expressed by these matrices for each section connected in cascade. Furthermore, this method makes it easily possible to compute with the constant power load (refer to Reference (2)).

### Analysis of CC Feeding Circuits

Figs. 1 and 2 show the circuit diagram and structural assemble of the CC feeding system, respectively. Fig. 3 shows the cross-section and characteristics of CC. To make a

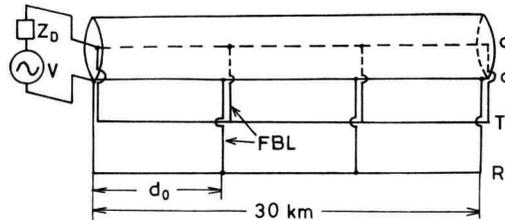


Fig. 1 The circuit diagram of power coaxial cable (CC) feeding circuits.  
 T; trolley wire, R; rail, c; the inner conductor of CC, d; the outer conductor of CC, FBL; feeding branch line,  $d_0$ ; the interval between FBLs, V; supply voltage 30 (kV),  $Z_D$ ; the internal impedance of power source  $j2(\Omega)$ .

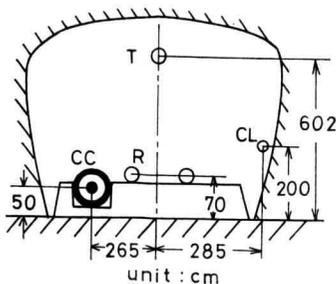
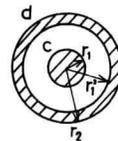


Fig. 2 The structural assembly of CC feeding circuits.  
 CL; communciation line, T, R and CC are same as in Fig. 1.



$r_1 = 2.855 \text{ cm}$   
 $r_1' = 4.305 \text{ cm}$   
 $r_2 = 5.200 \text{ cm}$   
 resistance of conductor (DC  $20^\circ\text{C}$ )  
 $= 0.00932 (\Omega/\text{km})$

Fig. 3 The cross-section and characteristics of power coaxial cable.

good use of the features of this system, the data of tunnel section are used for the structural assembly. The other data are as follows:

- earth conductivity;  $\sigma=0.01$  S/m,
- frequency;  $f=60$  Hz,
- load power;  $P=18$  MW (estimating a car speed at 210 km/hour),
- power factor of the load;  $\cos \varphi=0.8$ ,
- leakage resistance for earth of rail;  $R_g = 10 \Omega \cdot \text{km}$ .

The self impedance of each conductor and the mutual impedance between conductors are obtained by Carson-Pollaczek equations, since the line is a circuit returning through earth. (3)-(5) The parts of line in a system are divided into equal section lengths of 500 m, and each section is treated as a lumped constant circuit (refer to Reference (1)).

In the analysis, a constant power must be supplied so that the load may move at a constant speed. Therefore, the load impedance varies with its position. Provided that power factor of the load is constant in this computation, the load impedance is expressed by

$$Z_L = (24 + j18)/\alpha \quad (\Omega) \quad (1)$$

The value of  $\alpha$  is obtained by numerical analysis for the load power, that is, the value of  $\alpha$  is determined so that the value of calculated load power is within  $\pm 0.5\%$  of a constant power.

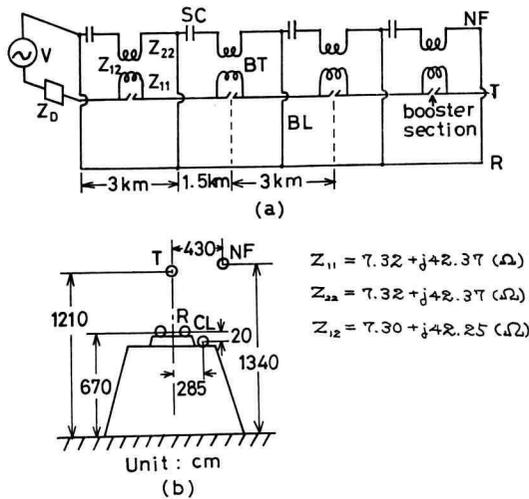


Fig. 4 BT feeding circuits. (a) circuit diagram; (b) structural assembly. NF; return (negative) feeder, T; trolley, R; rail, BT; booster transformer, SC; series condenser  $-j2(\Omega)$ , BL; boosting line, CL; communication line, V; supply voltage 30(kV),  $Z_D$ ; the internal impedance of power source  $j0.4 (\Omega)$ .

### Analysis of BT or AT (AT1 or ATa) Feeding Circuits

The results for CC feeding system are compared with the results for the other feeding systems. Then, the analysis of a feeding system using BT or AT is carried out in the same way. In the analysis, the same conditions for the CC feeding system should be used as many as possible. However, to make a good use of the features of respective types of systems, the data actually employed or examined in the feeding system in these computations<sup>(6)</sup>; i.e., the data of the standard overhead section in the New Tokaido Trunk Line are used for the BT feeding system, the data of the standard overhead section in the New Sanyo Trunk Line for the AT1 feeding system, and the estimated data of the overhead section for the ATa feeding system. Figs. 4(a) and (b) show the circuit diagram and structural assembly of the BT feeding system, respectively. The circuit diagram of the AT feeding system is shown in Fig. 5(a), the structural assembly of the AT1 feeding system in Fig. 5(b), and an example of the structural assembly of the ATa feeding system in Fig. 5(c).

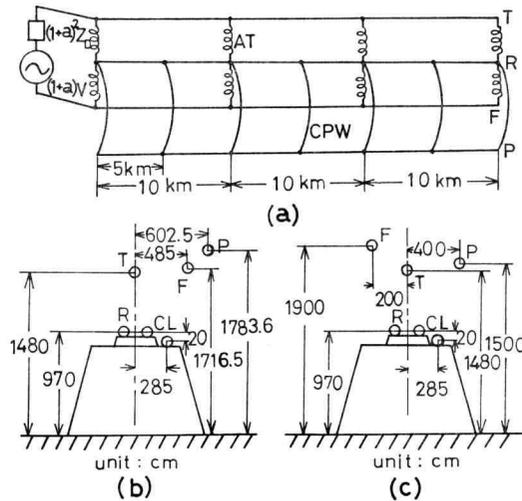


Fig. 5 AT feeding circuits. (a) circuit diagram; (b) structural assembly of the AT1 feeding system; (c) structural assembly of the ATa feeding system ( $a=1.5, 2.0$  or  $3.0$ ).

T; trolley, R; rail, F; positive feeder, P; protective wire, CL; communication line, AT; auto-transformer,  $a$ ; turn-ratio of AT, CPW; connector of protective wire,  $(1+a)^2 Z_D$ ; the internal impedance of power source  $Z_D=j2(\Omega)$ .

### Results of Analyses and Discussions

#### 1. Power Feeding Characteristics

Figs. 6-8 show the characteristics of terminal voltage and current of load in each method of feeding system. In the BT feeding system, the voltage drop is very large, because the booster transformer is inserted in a series with the line. Though the series condenser are inserted to improve the characteristics, it seems impossible to realize a feeding

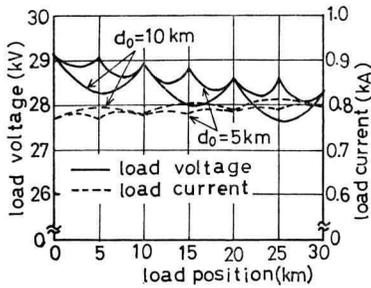


Fig. 6 Power feeding characteristics in the CC feeding system.

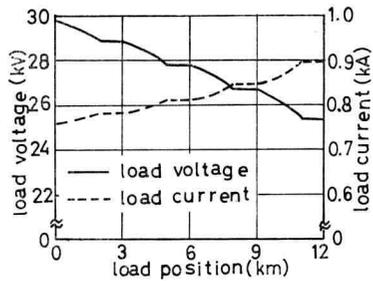


Fig. 7 Power feeding characteristics in the BT feeding system.

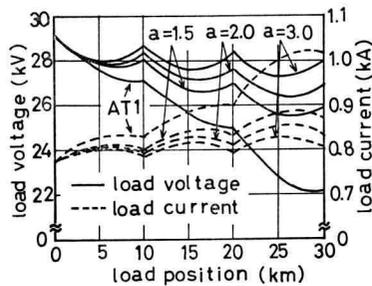


Fig. 8 Power feeding characteristics in the AT feeding system.

longer than about 15 km. In the AT1 feeding system, the voltage drop is so large that minimum voltage is approximately 22.5 kV. However, the voltage drop is improved if the value of turn-ratio,  $a$ , becomes large. In the CC feeding system, the result shows excellent characteristics of very small voltage drop, because of small reactance and power factor improvement owing to large capacitance between cable conductors. Therefore, it will be possible to realize a feeding longer than 30 km. Then, it is useful to compute such a CC feeding system that the total length of a feeding section is 60 km. The result is shown in Fig. 9. In this case as well as that of 30 km, the voltage drop is very small, that is, the minimum voltage of load is about 27 kV. This result shows it possible to include two loads in the same feeding section or to realize a feeding longer than 60 km in the case of only one load. The characteristics of load current show a trend just opposite to those of load terminal voltage, because of the calculation at a constant power. The AT1 feeding system requires current of more than 1000 A. On the other hand, the maximum of load current is only about 830 A as a result of the CC feeding system of Fig. 9. This is advantageous for the characteristics of an electromagnetic induction which is described in next section.

## 2. Electromagnetic Induction Characteristics

Figs. 10–12 show the induction characteristics of each feeding system. The total ampere-kilometer (Amp·km) is a basis of induction calculation of an induced circuit which is laid parallel to and apart from the feeding circuits throughout the length (refer to Appendix 1).

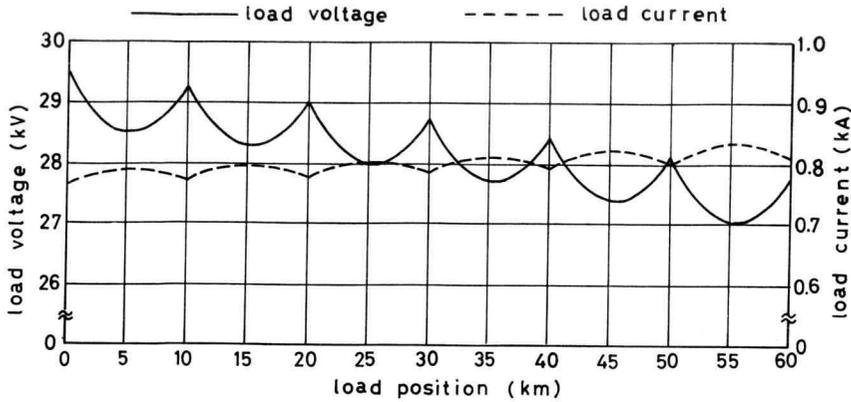


Fig. 9 Power feeding characteristics in the CC feeding system when the total length of a feeding section is 60 km.  $d_0 = 10$  km

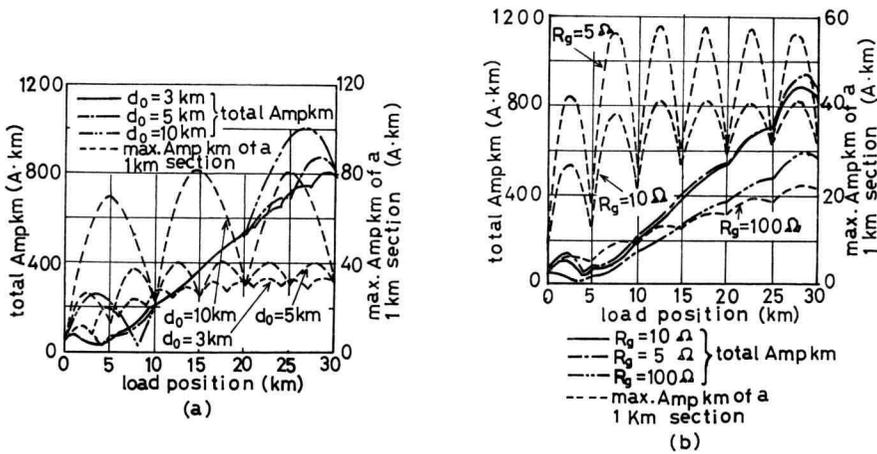


Fig. 10 Induction characteristics in the CC feeding system. (a) parameter  $d_0$ ; (b) parameter  $R_g$ .

While, the maximum Amp·km of a 1 km section is a basis of induction calculation of an induced circuit which is laid parallel to the feeding circuits at a part of the length (refer to Appendix 2). Figs. 10–12 show that leakage resistance for earth of rail,  $R_g$ , has much effect upon induction characteristics. When  $R_g$  is large, the inductive interference to the outside is small because of small rail-to-earth leakage current. Too large a value of  $R_g$ , however, will cause the other troubles because of high rail potential. The CC feeding system shows good characteristics, absorbing the rail current to the other conductor of CC, because of large mutual inductance between cable conductors, that is,  $Z_{cd} \approx Z_{dd}$ . Particularly, the maximum Amp·km of a 1 km section can be suppressed to a very small value by shortening interval between feeding branch lines. Fig. 13 shows the characteristic of the maximum Amp·km of a 1 km section in the case that the total length of a feeding section is 60 km.

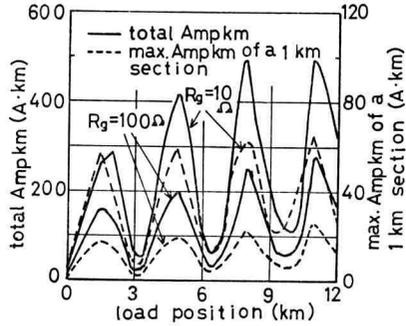


Fig. 11 Induction characteristics in the BT feeding system.

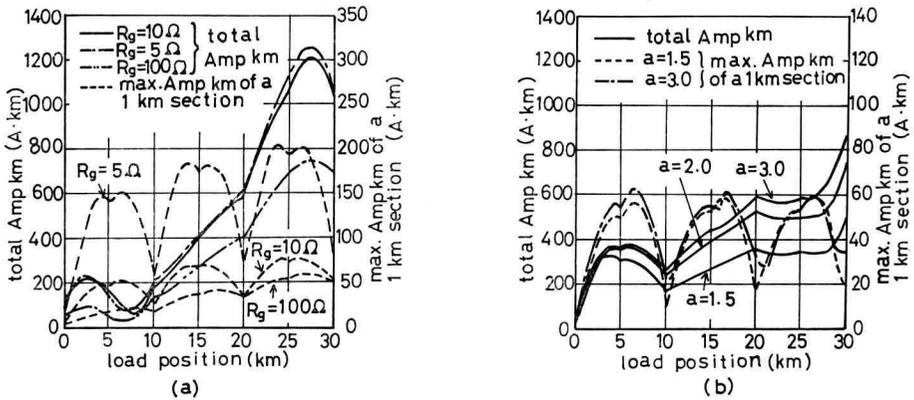


Fig. 12 Induction characteristics in the AT feeding system.  
(a) parameter  $R_g$ ; (b) parameter  $a$ .

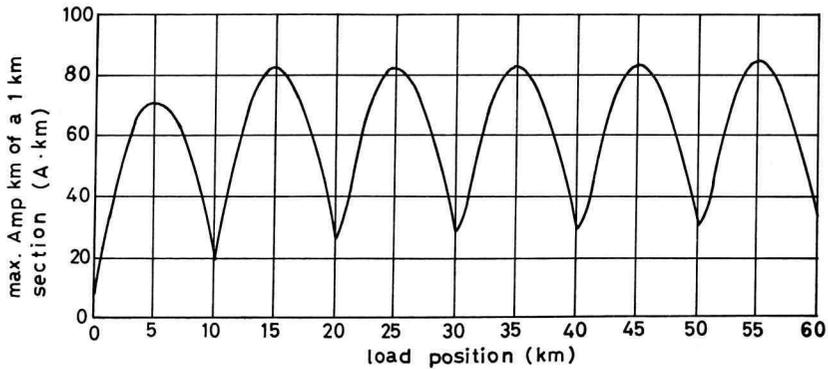


Fig. 13 Characteristic of the maximum Amp·km of a 1 km section in the CC feeding system when the total length of a feeding section is 60 km.  $d_0=10$  km

The maximum value of this result is about 80 Amp·km similar to the result of Fig. 10. Therefore, the results of Figs. 9 and 13 show that it is possible to realize a feeding system of fairly long length in the CC feeding method. In the BT feeding system, the maximum value of the total Amp·km is 500 A·km, which is very small compared with the other feeding system owing to the short feeding length of 12 km. In the AT feeding system, good characteristics regarding total Amp·km are obtained at  $a=1.5$ , while the maximum Amp·km of a 1 km section does not greatly differ with turn-ratios. In the AT1 feeding system, the induction is considerably large because of large load current.

Figs. 14–16 show the induced voltage in the induced circuit, CL (refer to Figs. 2, 4(b) and 5(b)), which exists parallel to and near from the feeding circuits throughout the length. The induced voltage may be calculated from the following equation;

$$V_c = \sum_j I_j Z_{jc} \quad (V), \quad (2)$$

where  $Z_{jc}$  is the mutual impedance between the  $j$ -th conductor of the feeding circuits and the induced circuit, CL. The actual induced voltage multiplies this value by an overall shielding factor, taking account of the effect of the shielding layer of communication cable, the shielding effect of overhead or tunnel section, and the influence of double-track line. Therefore, Figs. 14–16 show the induced voltage characteristics without considering these shielding

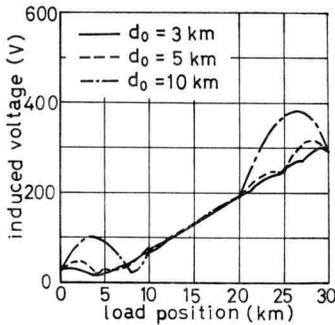


Fig. 14 Induced voltage characteristics in the CC feeding system.

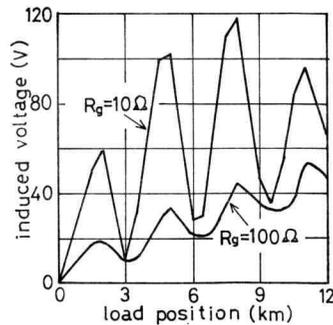


Fig. 15 Induced voltage characteristics in the BT feeding system.

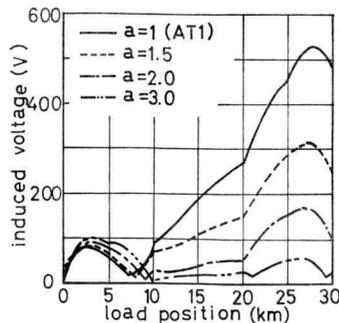


Fig. 16 Induced voltage characteristics in the AT feeding system.

effects. The BT feeding system shows excellent characteristics with very low induced voltage, because the feeding length is short and BT absorbs the rail current to the return wire (NF). The result of the CC feeding system shows better than that of AT1 feeding system. The maximum induced voltage of the former is about 380 V, while that of the latter is about 530 V.

3. The Distribution of Rail Potential and Earth Return Current

Figs. 17 and 19 show the rail potential and earth return current in the CC and AT feeding systems where the load is situated at 28 km from the sending end (substation). In both systems, the rail potential is maximum at the load position, and the maximum value is

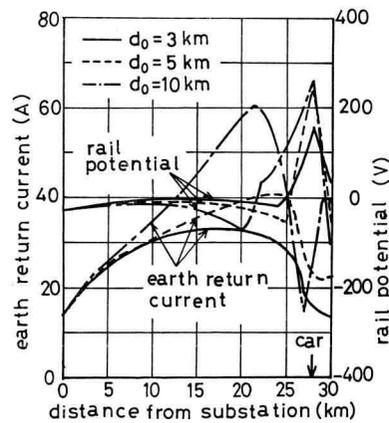


Fig. 17 The distributions of earth return current and rail potential in the CC feeding system where the load is situated at 28 km from the substation.

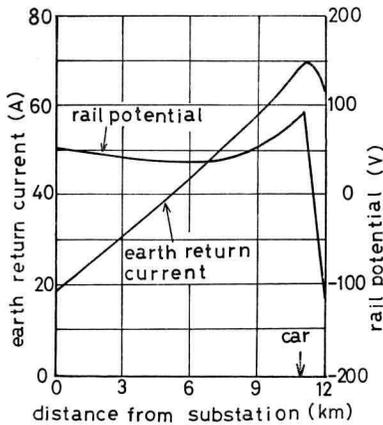


Fig. 18 The distributions of earth return current and rail potential in the BT feeding system where the load is situated at 11 km from the substation.

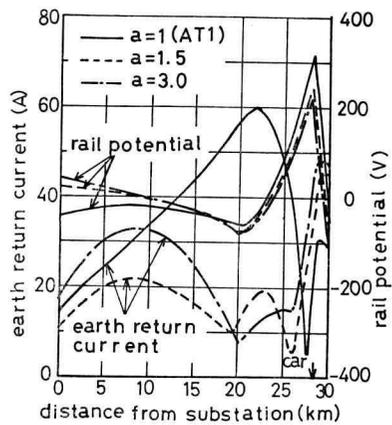


Fig. 19 The distributions of earth return current and rail potential in the AT feeding system where the load is situated at 28 km from the substation.

approximately 250 V. However, the rail potential in the CC feeding system may be reduced by shortening the interval between feeding branch lines. The earth return current is small at the load position, because it changes the direction at the position. Therefore, the position of the maximum value of earth return current is about 5 km on the sending-end side from the load position. These two characteristic values in the BT feeding system are small as shown in Fig. 18.

#### 4. Comparisons of Each Feeding Method

From these results, the merits and demerits of each feeding method are described as follows.

In the BT feeding system, though the induction characteristics are excellent, the power feeding characteristics are not desirable. This is because the voltage drop is large owing to booster transformer inserted in a series with the conductors, i.e., trolley wire and return feeder. Therefore, the characteristics must be improved by either shortening the feeding distance or inserting a series condenser in order to reduce the voltage drop. As seen in Fig. 4(a), a booster section must be provided at every booster transformer position. On passing the booster section, a load produces arc which is apt to damage the pantagraph. Especially, in high speed railroad, it is difficult to extinguish the arc once produced because of large current. Although a variety of arc extinguishing equipments have been devised, they are not yet perfect. Therefore, the BT feeding system is not suitable for high speed railroad.

In the AT feeding system using unity turn-ratio auto-transformer which has been put into practice in Japan, both the voltage drop and load current are large at a point near the nonsending end (sectioning post), and the induction characteristics are also not good. The system, however, makes it possible to move a large capacity at high speed, because there is not such a booster section as in the BT feeding system. Consequently, the AT1 feeding system is now the main a.c. feeding system in Japan. In the AT feeding system with auto-transformer turn-ratio  $a=1.5, 2.0$  or  $3.0$ , both the power feeding and induction characteristics are excellent, and this system has the possibility of realizing long distance feeding. This system, however, involves the difficult problems of feeding structure and apparatus insulation because of high feeding voltage. Moreover, it has the disadvantages that the auto-transformer needs large space, and that there is technical and economical limitation in reducing the leakage impedance.

In the CC feeding system, both the power feeding and induction characteristics are excellent. The characteristics of this system become better by shortening the interval between feeding branch lines. Particularly, the induction characteristics become very excellent. Furthermore, the rail potential may be reduced by shortening the interval. This system has the advantages that neither the booster section as in the BT feeding system nor the large space as in the AT feeding system is necessary. The results of this system shows it possible to move a large capacity at high speed and to realize feeding distance longer than 30 km (for example, 60 km as shown in Figs. 9 and 13). Therefore, this feeding system using power coaxial cable is quite suitable for the regions where the power supply

condition is not good, such as a long tunnel section or a remote and secluded place in the mountains, and where the electromagnetic inductive interference arising from this system becomes serious problem, such as a large city. However, in putting this system to practice use, the following two problems must be solved. Firstly, there is a risk of resonance phenomena between the capacitance of cable and the inductance from substation to load, owing to the harmonics generated at load. Secondly, there are the problems of temperature rise of cable, and of the connection at the position of feeding branch lines.

### Conclusions

This paper proposes the new a.c. feeding method using power coaxial cable, and the analytical results for this system are shown compared with the results for other feeding systems. The results for the CC feeding system show excellent characteristics, that is, a very small drop due to small reactance and good power factor improvement owing to the large capacitance between cable conductors. This feeding system also shows good induction characteristics in which the rail current is absorbed to the outer conductor of CC, because of large mutual inductance between cable conductors. Furthermore, it is clarified that shortening the interval between feeding branch lines results in better characteristics. Therefore, the CC feeding system is expected to be put to practical use.

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### Appendix 1. Total Amp·km

When the induced circuit is laid parallel to the feeding circuits throughout the length, the electromagnetic inductive voltage of the induced line without considering shielding effects is obtained from the following equation.

$$V = \left| \int_0^l \left( \sum_{k=1}^n Z_k I_k \right) dl \right|,$$

where  $Z_k$ ; the mutual impedance per km between the k-th conductor of the feeding circuits and the induced circuit ( $\Omega/\text{km}$ ),

$I_k$ ; the current flowing the k-th conductor of the inducing (feeding) circuits (A),

$l$ ; total length of a feeding section (km), and

$n$ ; the number of the conductors of the inducing circuits.

If the induced line exists fairly apart from the inducing circuits, the following assumption is given.

$$Z_1 = Z_2 = Z_3 = \dots = Z_n = Z$$

Thus, the electromagnetic voltage is obtained from the following equation.

$$V = |Z| \cdot \text{total Amp} \cdot \text{km}$$

$$\text{where total Amp} \cdot \text{km} = \left| \int_0^l \left( \sum_{k=1}^n I_k \right) dl \right|$$

Therefore, if the total Amp·km of the feeding system is known, the electromagnetic voltage of the induced circuit which is laid parallel to and apart from the feeding circuits throughout the length is easily obtained by asking the mutual impedance per km between the inducing and induced circuits.

## Appendix 2. Maximum Amp·km of a 1 km Section

When the induced circuit is laid parallel to and fairly apart from the feeding circuits at a part of the length,  $l'$ , the electromagnetic inductive voltage of the induced line without considering shielding effects is obtained from the following equation.

$$V' = |Z| \cdot \left| \int_0^{l'} \left( \sum_{k=1}^n I_k \right) dl \right|,$$

where the descriptions are the same as in Appendix 1. In this equation, the value of  $\sum_{k=1}^n I_k$  is variable depending on the position at which the induced circuit exists. Therefore, considering the most serious condition for the induced circuit, the maximum value of  $\sum_{k=1}^n I_k$  for a 1 km is taken as the inducing current. Then, we obtain

$$\text{maximum Amp} \cdot \text{km of a 1 km section} = \max. \left| \sum_{k=1}^n I_k \right|.$$

Thus, the electromagnetic voltage is obtained by the following equation.

$$V' \leq |Z| l' \cdot (\text{maximum Amp} \cdot \text{km of a 1 km section})$$

Therefore, if the maximum Amp·km of a 1 km section of the feeding system is known, the approximate value of the electromagnetic voltage of the induced circuit which is laid parallel to and apart from the feeding circuits at a part of the length is easily obtained by asking the mutual impedance per km between the inducing and induced circuits.