A Chart Calculating Load Impedance Giving Reflection Coefficient

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Abstract

We know that Smith chart consists of two orthogonal families of circles of r=constant and x=constant of normalized load impedance Z_L/Z on (u, v) complex plane of reflection coefficient $R=u+jv$. It is very useful for consideration and calculation on transmission line technics.

Now we have found a new chart calculating load impedance $Z_L=Z(r+jx)$ giving reflection constant K and phase angle ψ of reflection coefficient $R=-Ke^{j\psi}$. This chart consists of two orthogonal families of circles of K=constant and ψ=constant on (r, x) complex plane of normalized load impedance $Z_L/Z=r+jx$. It is rather familiar using $S=(1+K)/(1-K)$ instead of K.

This chart is useful for consideration and calculation on transmission line technics, and also available for the case of wave guide and plane wave propagating on to a semi-infinite medium or slab.

1. Introduction

We will consider a trasmission line of characteristic impedance Z terminated with load impedance Z_L (Fig. 1a). A plane wave propagating toward right along the transmissoin line will be partially reflected at the terminal, therefore, we may observe voltage standing wave pattern like as shown in Fig. 1b. Reflection coefficient R will be given by

$$R=\frac{Z_L-Z}{Z_L+Z}=\frac{Z_L/Z-1}{Z_L/Z+1}=u+jv \tag{1}$$

$$=-Ke^{j\psi}, \tag{2}$$

where $Z_L/Z=r+jx$ is the normalized impedance of load.

$$S=\frac{1+K}{1-K} \tag{3}$$

is called as voltage standing wave ratio. We also have the relation

$$\frac{\psi}{2}=\frac{2\pi d}{\lambda}$$

referring to Fig. 1b.

We know the fact that Smith chart[1] consists of two orthogonal families of circles of r=constant and x=constant on (u, v) complex plane of reflection coefficient $R=u+jv$.

On the other hand, we found the new chart calculating load impedance $Z_L=Z(r+jx)$ giving reflection constant K and phase angle ψ of the reflection coefficient $R=-Ke^{j\psi}$. This chart consists of two orthogonal families of circles of K=constant and ψ=constant on (r, x)

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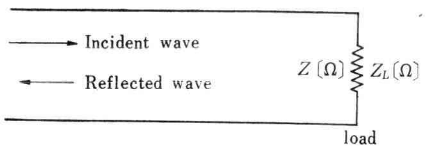
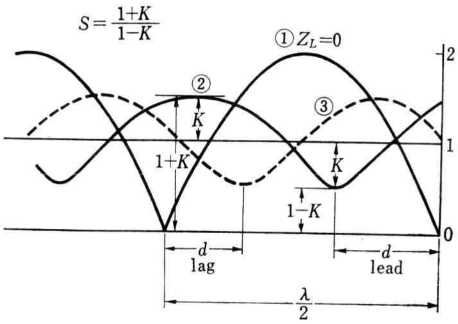


a.



b.

Fig. 1 a. Transmission line with characteristic impedance $Z\,[\Omega]$ is terminated with load $Z_L\ [\Omega]$.
b. Voltage standing wave pattern on transmission line.
① $Z_L=0,\ \ \psi=0,$

② Z_L is finite, $\dfrac{\psi}{2}=\dfrac{2\pi d}{\lambda}>0,$

③ Z_L is finite, $\dfrac{\psi}{2}=\dfrac{2\pi d}{\lambda}<0.$

complex plane of normalized load impedance $Z_L/Z=r+jx$. This new chart is very useful for the measurement of normalized load impedance $r+jx$ or load impedance $Z(r+jx)$ giving either reflection constant K or voltage standing wave ratio S and phase angle ψ.

This chart is useful for consideration and calculation on transmission line techninics, and also available for the case of wave guide and propagating plane wave normally on a semi-infinite medium or slab.

2. Theory and chart

From [2], we have the relation:

$$\frac{Z_L}{Z}=\frac{1-Ke^{j\psi}}{1+Ke^{j\psi}}=\frac{e^{j(\psi/2)}-Ke^{-j(\psi/2)}}{e^{j(\psi/2)}+Ke^{-j(\psi/2)}}. \qquad [4]$$

Using Euler's formula and [3], and rationalizing the denominator,

$$=\frac{S\left(1+\tan^2\dfrac{\psi}{2}\right)+j(1-S^2)\tan\dfrac{\psi}{2}}{S^2+\tan^2\dfrac{\psi}{2}}.$$

Therefore,

$$r=\frac{S\left(1+\tan^2\dfrac{\psi}{2}\right)}{S^2+\tan^2\dfrac{\psi}{2}}, \tag{5}$$

$$x=\frac{(1-S^2)\tan\dfrac{\psi}{2}}{S^2+\tan^2\dfrac{\psi}{2}}. \tag{6}$$

From [5] and [6], we have the relation:

$$Sr+x\tan\frac{\psi}{2}=1. \tag{7}$$

Now, we will assume $r\neq 0$ and $x\neq 0$,

$$\tan\frac{\psi}{2}=\frac{1-Sr}{x}, \tag{8}$$

$$S=\frac{1-x\tan\dfrac{\psi}{2}}{r}. \tag{9}$$

From [5] and [6], we also have

$$\frac{S\left(1+\tan^2\dfrac{\psi}{2}\right)}{r}=\frac{(1-S^2)\tan\dfrac{\psi}{2}}{x}. \tag{10}$$

Introducing [8] into [10] and calculating this, we finally have

$$\left\{r-\frac{1}{2}\left(S+\frac{1}{S}\right)\right\}^2+x^2=\left\{\frac{1}{2}\left(S-\frac{1}{S}\right)\right\}^2. \tag{11}$$

Introducing [9] into [10] and calculating this, we finally have

$$r^2+\left\{x+\frac{1}{2}\left(\tan\frac{\psi}{2}-\frac{1}{\tan\dfrac{\psi}{2}}\right)\right\}^2=\left\{\frac{1}{2}\left(\tan\frac{\psi}{2}+\frac{1}{\tan\dfrac{\psi}{2}}\right)\right\}^2. \tag{12}$$

[11] and [12] are both the equations of circles and make an orthogonal set.

If we put [3] into [11], we have

$$\left(r-\frac{1+K^2}{1-K^2}\right)^2+x^2=\left(\frac{2K}{1-K^2}\right)^2. \tag{13}$$

We may take either [11] and [12] or [13] and [12]. Fig. 2 shows the chart using [11] and [12], supplemented by [13].

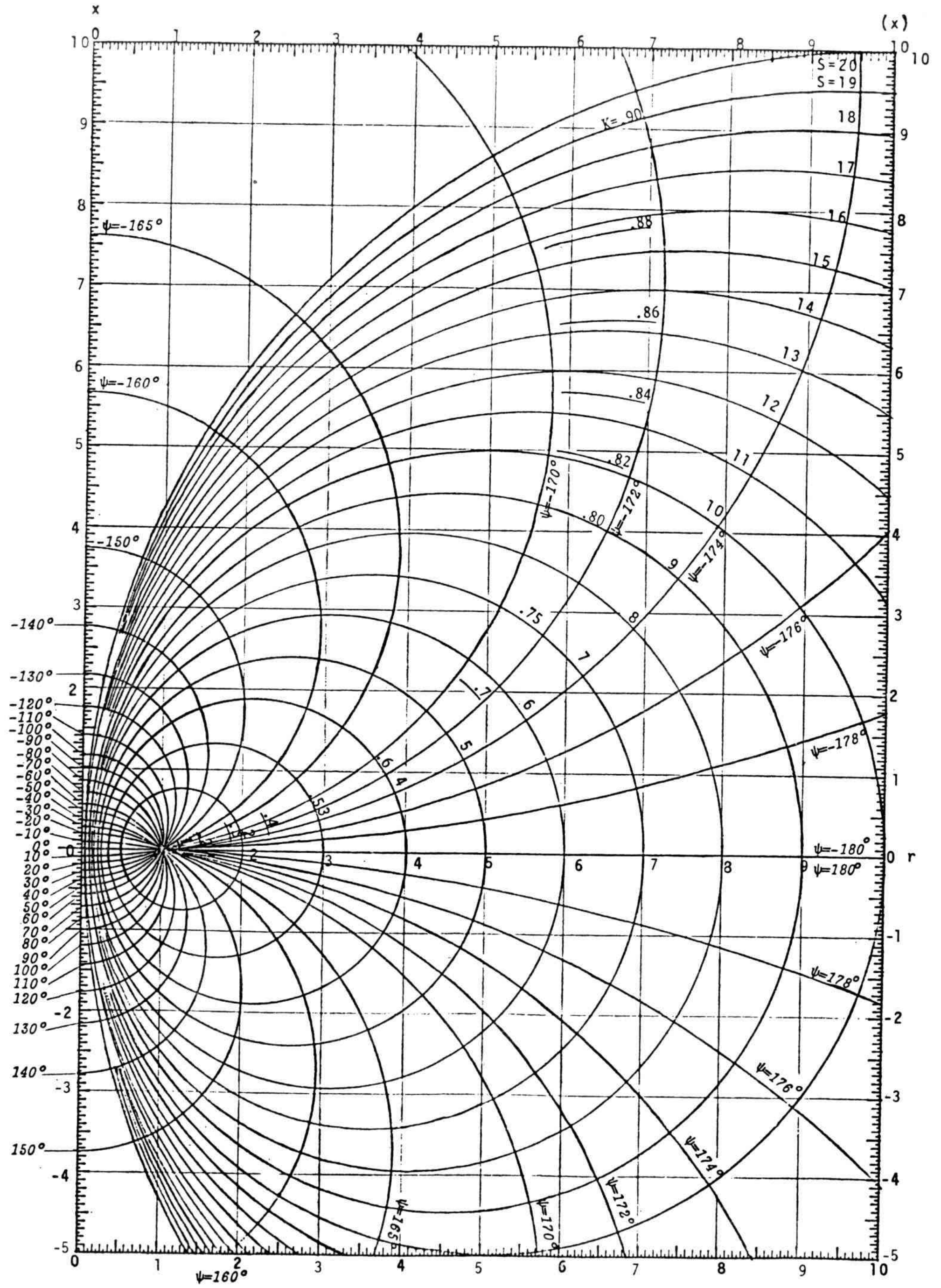


Fig. 2 The new chart calculating load impedance giving reflection coefficient.

3. Calculating examples

Example 1. $Z=300\,\Omega$ is known, and $K=0.75$, $\psi/2=-80°$ are measured. Calculate the load impedance.

$S=\dfrac{1+K}{1-K}=7.0$, $\psi=-160°$, we have the values

$$r+jx=2.86+j\,3.35$$

from the chart.

$$\therefore Z_L=Z\,(r+jx)=860+j\,1000\,\Omega$$

Example 2. $Z=600\,\Omega$ is known, and $K=0.6$, $\psi/2=15°$ are measured. Calculate the load impedance.

$S=\dfrac{1+K}{1-K}=4.0$, $\psi=30°$, we have the values

$$r+jx=0.27+j\,0.25$$

from the chart.

$$\therefore Z_L=Z\,(r+jx)=160-j\,150\,\Omega$$

References

1) Murakami, I.: "Mathematics for electromagnetic theory", Hirokawa, *p.* 139 (1976).