

Earth-return Mutual Impedance between Overhead Power Line and Underground Communication Line (when the two lines are separated by a large distance)

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Abstract

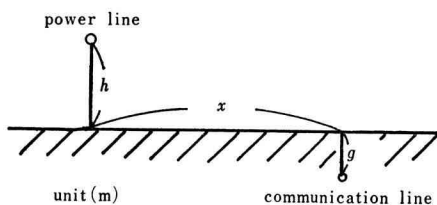
An analytical expression for the earth-return mutual impedance between overhead power line and underground communication line is given only in an integral form as shown in the papers of Carson and Pollaczek. However, a practical method enabling an approximate numerical evaluation of this form seems not to have been introduced as yet. From the reason just mentioned, the present authors develop in this paper the integral form to an infinite series by applying Watson's asymptotic expansion theorem⁽³⁾. In addition, it is confirmed that an approximate value satisfying practical purposes of the mutual impedance is obtained by computing the five leading terms of the infinite series in cases in which the two lines are separated by a large distance.

1. Introduction

It is widely known that the earth-return mutual impedance between two overhead lines is calculated by use of Carson-Pollaczek's equations^{1),2)}. In addition, the earth-return mutual impedance between overhead power line and underground communication line is given by Carson-Pollaczek, but, only in an integral form^{1),2)}, and a practical method enabling an approximate evaluation of this form seems not to have been developed. The authors therefore attempted to develop the above-mentioned practical method.

2. Integral form

The earth-return mutual impedance between the two lines shown in Fig. 1 is given by the following definite integral



$$g' = g \sqrt{\alpha}, h' = h \sqrt{\alpha}, x' = x \sqrt{\alpha}$$

$$\alpha = \omega \sigma \times 4\pi \times 10^{-7}$$

$$\omega = 2\pi f$$

σ : the earth conductivity

Fig. 1. Geometry for Eq. (1).

$$Z_m = 4\omega(P + jQ) \times 10^{-4} = 4\omega \times 10^{-4} \int_0^{\infty} (\sqrt{\mu^2 + j} - \mu) \cos x' \mu e^{-(h' \mu + g' \sqrt{\mu^2 + j})} d\mu \quad (1)$$

where g' , h' and x' stand for the physical parameters as shown in Fig. 1, and P and Q are real, respectively. In addition, r and θ are defined as follows,

$$r = \sqrt{x'^2 + h'^2}, \quad \theta = \tan^{-1}(x'/h').$$

3. Infinite series form

If r is large enough, namely, when the two lines are separated by a large distance, then by Watson's theorem (refer to Appendix) the asymptotic expansion for r of equation (1) leads to the following equation

$$P + jQ = \sum_{n=0}^{\infty} \left\{ \left(\sum_{k_1+k_2=n} F(k_1)G(k_2) \right) \frac{(2n)!}{e^{g'F(0)}} \frac{\cos(2n+1)\theta}{r^{2n+1}} - G(n) \frac{(2n+1)!}{e^{g'F(0)}} \frac{\cos(2n+2)\theta}{r^{2n+2}} \right\} \quad (2)$$

where k_1 and k_2 are non-negative integers and $F(0) = \sqrt{j}$, $F(n) = (1/n!) \times (1/2) \times \cdots \times \left(\frac{1}{2} - n + 1\right) \sqrt{j}/j^n$, $G(0) = 1$, $G(n) = \sum_{l=1}^n (-1)^n g'^n \sum_{k_1+k_2+\cdots+k_l=n} F(k_1)F(k_2)\cdots F(k_l)/n!$, and k_l is a positive integer.

The computed results of P and Q are shown in Fig. 2 and Fig. 3 for the case of $r=6$ and 10

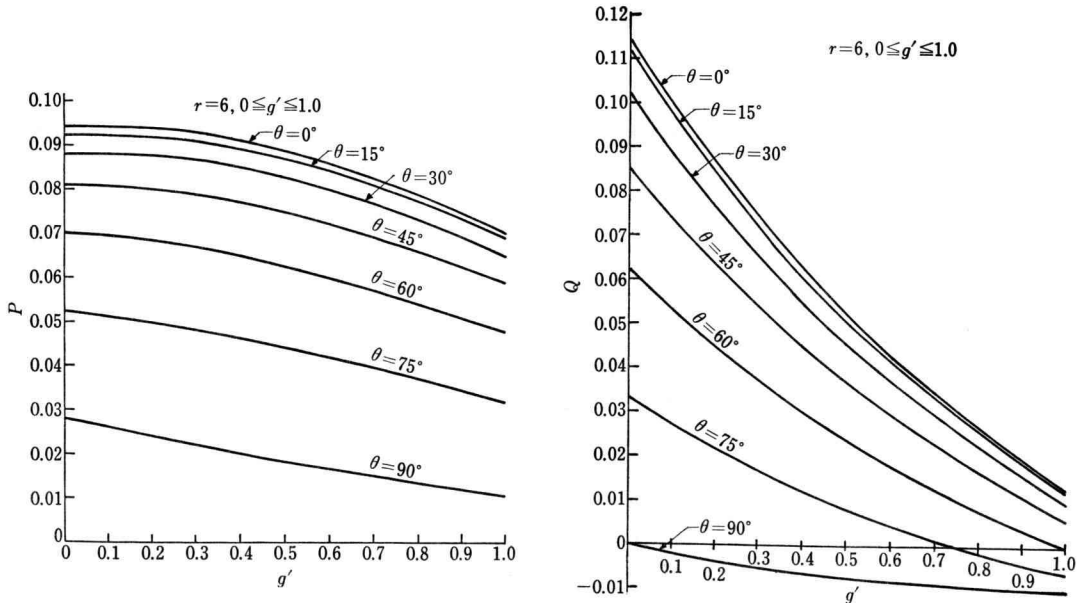


Fig. 2. Computed results of P , Q ($r=6$).

and $0 \leq g' \leq 1$. For $\theta = 90^\circ$, Q becomes negative. It is interpreted that this phenomenon arises from the effect which is stated as follows: the underground communication line interlinks with the magnetic flux arising from the earth-return current to a greater extent than that arising from the current in the overhead power line. However, it is required to consider this point in more detail by investigating the distribution of the earth-return current.

Now, the five leading terms (from $n=0$ to $n=4$) of Eq. (2) are given by the following equations

$$\begin{aligned}
 P = & \frac{1}{\sqrt{2} e^{g'/\sqrt{2}}} \left(\cos \frac{g'}{\sqrt{2}} + \sin \frac{g'}{\sqrt{2}} \right) \frac{\cos \theta}{r} - \frac{1}{e^{g'/\sqrt{2}}} \cos \frac{g'}{\sqrt{2}} \frac{\cos 2\theta}{r^2} \\
 & + \frac{1}{e^{g'/\sqrt{2}}} \left\{ \left(\frac{1}{\sqrt{2}} - g' \right) \cos \frac{g'}{\sqrt{2}} - \frac{1}{\sqrt{2}} \sin \frac{g'}{\sqrt{2}} \right\} \frac{\cos 3\theta}{r^3} \\
 & + \frac{3g'}{e^{g'/\sqrt{2}}} \left(\frac{1}{\sqrt{2}} \cos \frac{g'}{\sqrt{2}} - \frac{1}{\sqrt{2}} \sin \frac{g'}{\sqrt{2}} \right) \frac{\cos 4\theta}{r^4} \\
 & + \frac{3}{e^{g'/\sqrt{2}}} \left\{ \cos \frac{g'}{\sqrt{2}} \left(\frac{\sqrt{2} g'^2 + \sqrt{2}}{2} \right) + \sin \frac{g'}{\sqrt{2}} \left(g' - \frac{\sqrt{2}}{\sqrt{2}} g'^2 + \frac{\sqrt{2}}{2} \right) \right\} \frac{\cos 5\theta}{r^5}
 \end{aligned} \tag{3}$$

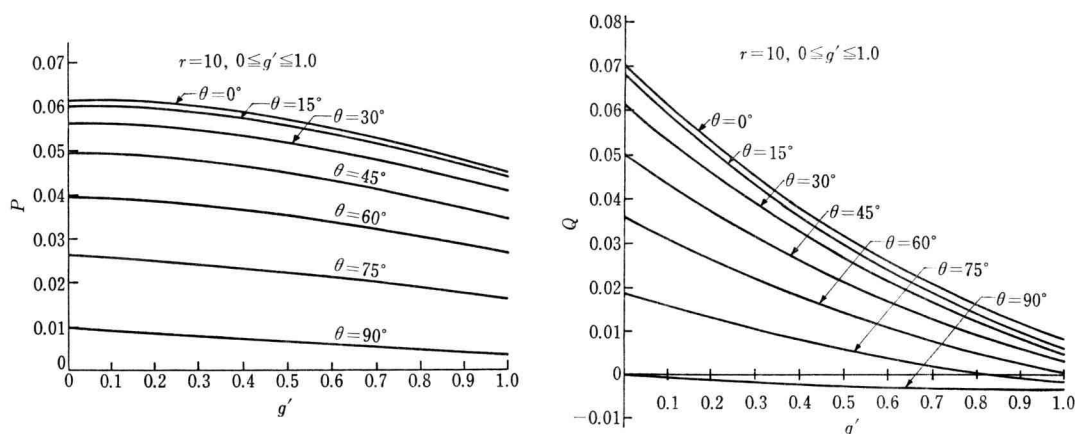
$$\begin{aligned}
 Q = & \frac{1}{\sqrt{2} e^{g'/\sqrt{2}}} \left(\cos \frac{g'}{\sqrt{2}} - \sin \frac{g'}{\sqrt{2}} \right) \frac{\cos \theta}{r} + \frac{1}{e^{g'/\sqrt{2}}} \sin \frac{g'}{\sqrt{2}} \frac{\cos 2\theta}{r^2} \\
 & - \frac{1}{e^{g'/\sqrt{2}}} \left\{ \frac{1}{\sqrt{2}} \cos \frac{g'}{\sqrt{2}} + \left(\frac{1}{\sqrt{2}} - g' \right) \sin \frac{g'}{\sqrt{2}} \right\} \frac{\cos 3\theta}{r^3} \\
 & - \frac{3g'}{e^{g'/\sqrt{2}}} \left(\frac{1}{\sqrt{2}} \sin \frac{g'}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cos \frac{g'}{\sqrt{2}} \right) \frac{\cos 4\theta}{r^4} \\
 & + \frac{3}{e^{g'/\sqrt{2}}} \left\{ \cos \frac{g'}{\sqrt{2}} \left(g' - \frac{\sqrt{2}}{2} g'^2 + \frac{\sqrt{2}}{2} \right) - \sin \frac{g'}{\sqrt{2}} \left(\frac{\sqrt{2} g'^2 + \sqrt{2}}{2} \right) \right\} \frac{\cos 5\theta}{r^5}
 \end{aligned} \tag{4}$$

4. Examination of Eqs. (3) and (4)

Let $g'=0$ in Eq. (2), then the result corresponds to the equations of the mutual impedance between a pair of overhead lines for $r \geq 6$ which was solved by Carson. Furthermore, the three leading terms of (3) and (4) correspond to the approximate equation of Reference (2) under the condition of $x, h \gg g$. In practical computation, Eq. (2) is approximated for large r by the five leading terms only. It is confirmed that the results of Eqs. (3) and (4) are kept within the difference of a few percents (Table 1) in comparison with the results from a numerical integration by Simpson's method. Accordingly, it is practically possible to compute P and Q by using Eqs. (3) and (4) for $r \geq 6$.

Table 1 Comparison of the approximate and the numerically integrated values

	r	θ	g'	Numerical integration	Approximate value	Relative error (%)
P	6	0	0	0.0935	0.0936	0.1
	6	0	0.1	0.0942	0.0943	0.1
	6	0	1.0	0.0705	0.0704	0.2
	6	90	0	0.0287	0.0278	3.3
Q	6	75	1.0	-6.045×10^{-3}	-6.080×10^{-3}	0.6

Fig. 3. Computed results of P , Q ($r=10$).

5. Conclusion

In this paper, the following points are confirmed;

(1) The earth-return mutual impedance between overhead power line and underground communication line is obtained, in cases in which the two lines are separated by a large distance, from the asymptotic expansion of the definite integral which was solved by Carson and Pollaczek.

(2) The approximate value is obtained for practical purposes by computing the five leading terms of the infinite series given by Eqs. (3) and (4).

When the two lines are separated by a small distance, it is required to work out a series which converges rapidly to the definite integral.

Acknowledgement

The authors would like to thank Dr. Akira KINASE, Prof. of Iktoku Technical University, for his valuable comments.

References

- (1) J.R. Carson: B.S.T.J. vol 5 p. 539 (1926-10)
- (2) F. Pollaczek: E.N.T. vol 3 p. 339 (1926-9)

(3) Ziro Yanagihara: "Progression" p. 227, Asakura (1975)

Appendix

[Watson's theorem] $f(t)$ is defined for $|t| < \rho$ (ρ is a constant.), where

$$f(t) = a_0 + a_1 t + a_2 t^2 + \dots$$

If $\int_0^A e^{-zt} t^\lambda |f(t)| dt < +\infty$, then

$$\int_0^A e^{-zt} t^\lambda f(t) dt \longrightarrow z^{-\lambda-1} \sum_{k=0}^{\infty} -\frac{a_k \Gamma(\lambda+k+1)}{z^k} \quad (x \rightarrow +\infty) \quad (30.10),$$

where $0 \leq A \leq +\infty$, $R_e Z = (x > 0)$ and $\lambda > 0$.

Especially, let $\lambda=0$, then

$$\int_0^A e^{-zt} f(t) dt \longrightarrow \sum_{n=0}^{\infty} \frac{n! a_n}{z^{n+1}} = \frac{a_0}{z} + \frac{a_1}{z^2} + \frac{2! a_2}{z^3} + \dots \quad (30.11).$$

(Proof) If R is defined by $\int_{\delta}^A e^{-zt} t^\lambda f(t) dt$ and δ is fixed as $0 < \delta < \rho$, then

$$|R| = \left| \int_{\delta}^A e^{-zt} t^\lambda f(t) dt \right| \leq \int_{\delta}^A e^{-zt} t^\lambda |f(t)| dt \leq K \cdot e^{-\delta x}, \quad (30.27)$$

K being a constant.

Now, let $f(t) = a_0 + a_1 t + \dots + a_n t^n + r_n(t)$, $r_n(t) = g(t) t^{n+1}/(n+1)!$, then

$$\int_0^{\delta} e^{-zt} t^\lambda f(t) dt = \sum_{k=0}^n a_k \int_0^{\delta} e^{-zt} t^{\lambda+k} dt + \int_0^{\delta} e^{-zt} t^\lambda r_n(t) dt.$$

Since $g(t)$ is continuous in $0 \leq t \leq \delta$, there exists A_n such that $|g(t)| \leq A_n$ in $0 \leq t \leq \delta$. Therefore,

$$|R_n| = \left| \int_0^{\delta} e^{-zt} t^{\lambda+n+1} g(t)/(n+1)! dt \right| \leq A_n/(n+1)! \int_0^{\delta} e^{-zt} t^{\lambda+n+1} dt.$$

Let $xt = u$, then

$$\begin{aligned} \left| \int_0^{\delta} e^{-xt} t^{\lambda+n+1} dt \right| &= \left| \int_0^{x\delta} e^{-u} u^{\lambda+n+2} du / x^{\lambda+n+2} \right| \\ &\leq \frac{1}{x^{\lambda+n+2}} \int_0^{\infty} e^{-u} u^{\lambda+n+1} du = \Gamma(\lambda+n+2) / x^{\lambda+n+2} \end{aligned}$$

Therefore $|R_n| \leq B_n / x^{\lambda+n+2}$, $B_n = A_n \Gamma(\lambda+n+2) / (n+1)!$ (30.29)

$$\lim_{x \rightarrow +\infty} x^{\lambda+n+1} R_n = 0.$$

Let $zt=u$, then

$$\begin{aligned} \int_0^{\delta} e^{-zt} t^{\lambda+k} dt &= z^{-\lambda-k-1} \int_0^{z\delta} e^{-u} u^{\lambda+k} du \\ &= z^{-\lambda-k-1} \left\{ \int_0^{\infty} e^{-u} u^{\lambda+k} du - \int_{z\delta}^{\infty} e^{-u} u^{\lambda+k} du \right\} \\ &= \Gamma(\lambda+k+1) z^{-\lambda-k-1} - \alpha_k, \end{aligned}$$

where

$$\alpha_k = z^{-\lambda-k-1} \int_{z\delta}^{\infty} e^{-u} u^{\lambda+k} du.$$

Let $u=zt$, then $\alpha_k = \int_{\delta}^{\infty} e^{-zt} t^{\lambda+k} dt$.

Therefore, $|\alpha_k| \leq \int_{\delta}^{\infty} e^{-xt} t^{\lambda+k} dt$. Further, letting $xt=u$,

$$\begin{aligned} |\alpha_k| &\leq x^{-\lambda-k-1} \int_{x\delta}^{\infty} u^{\lambda+k} e^{-u} du. \\ \therefore |x^{n+\lambda+1} \alpha_k| &\leq x^{n-k} \int_{x\delta}^{\infty} u^{\lambda+k} e^{-u} du \\ &\leq \delta^{k-n} \int_{x\delta}^{\infty} u^{n+\lambda} e^{-u} du. \end{aligned}$$

Since $\int_{x\delta}^{\infty} u^{n+\lambda} e^{-u} du \longrightarrow 0 \quad (x \rightarrow \infty), \quad \lim_{x \rightarrow \infty} x^{n+\lambda+1} \alpha_k = 0 \quad (k = 0, \dots, n) \quad (30.30)$

Therefore $\int_0^A e^{-zt} t^{\lambda} f(t) dt = \sum_{k=0}^n \frac{a_k \Gamma(\lambda+k+1)}{x^{\lambda+k+1}} - \sum_{k=0}^n \alpha_k + R_n + R$.

Eqs. (30.27), (30.29) and (30.30) lead to

$$\lim_{x \rightarrow \infty} x^{n+\lambda+1} \left\{ - \sum_{k=0}^n \alpha_k + R_n + R \right\} = 0.$$

Eq. (30.10) has thus been proved. (Q.E.D.)