Theoretical Considerations on another Contactless Measuring Methods of Electrical Conductivity of Semiconductor Circular Wafer

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Abstract

Authors derived an electromagnetic theory of the contactless measuring methods of electric conductivity of a semiconductor circular wafer measuring the change of input impedance of a coil with self-inductance L [H] when a semiconductor circular wafer approached to the coil from infinity keeping the situation shown in Fig. 1, which may be expressed as

$$Z = j\omega L + \frac{\pi^3}{8} f^2 \mu_0^2 \sigma a^4 n^2 t \frac{b^4}{d^2(d^2+b^2)^2} \qquad [\Omega]$$

under the condition $f \ll 50$MHz for $R \ll 1$ m as shown in a companion paper.[1]

For this measurement, we may use a high frequency Wheatston bridge, vector impedance meter or Q meter and so on. Here we will mainly explain the method using the high frequency Wheatston brige.

1. Calculation of impedance of a coil when a semiconductor circular wafer was placed in front of the coil.

Fig. 1 shows arrangement of the coil and the semiconductor circular wafer. Axial lines of the coil and the semiconductor circular wafer coincide with z-axis of a cartesian coordinates (x, y, z), and the center of the coil is on the origin 0 of the coordinates. The distance from 0 to the neutral plane of the wafer is d[m]. a[m] is the raidus of the coil, l[m] the length, n the number of turns of the coil; b[m] is the radius, t[m] the thickness of the semiconductor circular wafer.

When the coil is sufficiently small, we may assume that the coil is a magnetic dipole with a moment[2)]

$$\boldsymbol{p}_m = \boldsymbol{a}_z \mu_0 \pi a^2 I n, \qquad [\text{Wb m}] \tag{1}$$

where I[A] is the current in the coil.

We assume that the wafer is not magnetic, i.e., its permeability is μ_0. The coil with magnetic depole moment $\boldsymbol{p}_m = \boldsymbol{a}_z p_m$[Wb m] will make the magnetic field $\boldsymbol{H}(Q)$ [A/m] in the point Q on the neutral plane of the wafer[3)]

$$\boldsymbol{H}(Q) = \boldsymbol{a}_r \frac{p_m \cos\theta}{2\pi\mu_0 r^3} + \boldsymbol{a}_\theta \frac{p_m \sin\theta}{4\pi\mu_0 r^3} \qquad [\text{A/m}] \tag{2}$$

and then, magnetic flux density at that point would be

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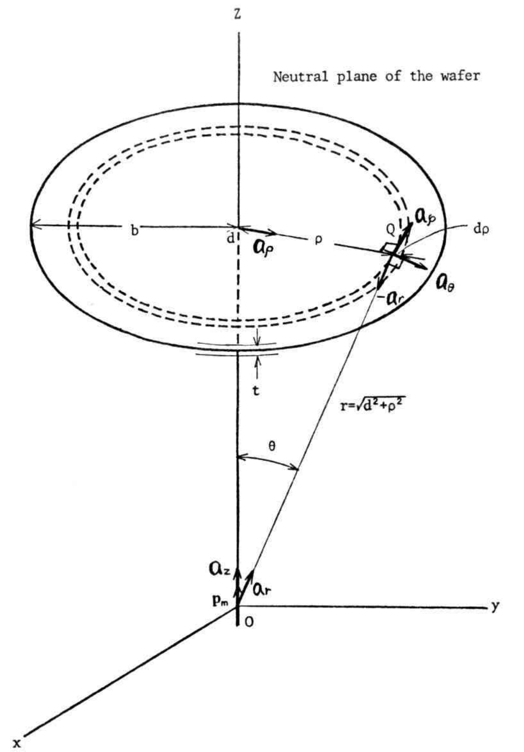

Fig. 1. Relation between the coil and the wafer.

$$\boldsymbol{B}(Q)=\boldsymbol{a}_r\frac{p_m\cos\theta}{2\pi r^3}+\boldsymbol{a}_\theta\frac{p_m\sin\theta}{4\pi r^3}. \qquad [\text{Wb/m}^2] \qquad (3)$$

The normal component of magnetic flux density passing upward through at the point Q is given by

$$B_z(Q)=\boldsymbol{a}_z\cdot\boldsymbol{B}(Q)=\boldsymbol{a}_z\cdot\boldsymbol{a}_r\frac{p_m d}{2\pi r^4}+\boldsymbol{a}_z\cdot\boldsymbol{a}_\theta\frac{p_m\rho}{4\pi r^4}=\frac{3p_m d^2}{4\pi r^5}-\frac{p_m}{4\pi r^3}. \quad [\text{Wb/m}^2] \qquad (4)$$

Total magnetic flux passing upward through a circle of radius ρ on the neutral plane of the wafer would be

$$\Phi_m(\rho)=\int_0^\rho B_z(Q)\,2\pi\rho\,d\rho=\frac{6\pi p_m d^2}{4\pi}\int_0^\rho\frac{\rho d\rho}{r^5}-\frac{2\pi p_m}{4\pi}\int_0^\rho\frac{\rho d\rho}{r^3}, \qquad [\text{Wb}] \qquad (5)$$

wher $d^2+\rho^2=r^2$; then, $\rho d\rho=rdr$; $\rho=0$, $r=d$; $\rho=\rho$, $r=\sqrt{d^2+\rho^2}$;

$$\Phi_m(\rho)=\frac{6\pi p_m d^2}{4\pi}\int_d^{\sqrt{d^2+\rho^2}}\frac{rdr}{r^5}-\frac{2\pi p_m}{4\pi}\int_d^{\sqrt{d^2+\rho^2}}\frac{rdr}{r^3}=\frac{p_m\rho^2}{2\sqrt{d^2+\rho^2}^{\,3}}. \quad [\text{Wb}] \qquad (6)$$

According to Faraday's law of induction, electromotive force $V(\rho)$ induced along a coaxial circle C of radius ρ on the neutral plane of the semiconductor circular wafer would be:

$$V(\rho) = -\frac{d\Phi_m(\rho)}{dt} = -j\omega\Phi_m(\rho) = -j\omega\frac{p_m\rho^2}{2\sqrt{d^2+\rho^2}^3}. \quad [\mathrm{V}] \quad (7)$$

Therefore electric field $\boldsymbol{E}(\rho)$ along C has to be[4]:

$$\boldsymbol{E}(\rho) = \boldsymbol{a}_\varphi\frac{V(\rho)}{2\pi\rho} = -\boldsymbol{a}_\varphi j\omega\frac{p_m\rho}{4\pi\sqrt{d^2+\rho^2}^3} = -\boldsymbol{a}_\varphi j\omega\frac{p_m\sin\theta}{4\pi r^2}. \quad [\mathrm{V/m}] \quad (8)$$

The electric current density $\boldsymbol{J}(\rho)$ in the semiconductor circular wafer along C would be:

$$\boldsymbol{J}(\rho) = \sigma\boldsymbol{E}(\rho) = -\boldsymbol{a}_\varphi j\omega\frac{\sigma p_m\rho}{4\pi\sqrt{d^2+\rho^2}^3}, \quad [\mathrm{A/m^2}] \quad (9)$$

and current surface density $\boldsymbol{K}(\rho)$ in the wafer is

$$\boldsymbol{K}(\rho) = \boldsymbol{J}(\rho)\,t = -\boldsymbol{a}_\varphi j\omega\frac{\sigma p_m\rho t}{4\pi\sqrt{d^2+\rho^2}^3}. \quad [\mathrm{A/m}] \quad (10)$$

According to Biot-Savart's law, magnetic field $\boldsymbol{H}(\rho)$ produced on 0 which is the center of the coil would be:

$$\boldsymbol{H}(o) = \frac{1}{4\pi}\int_s\frac{\boldsymbol{K}(\rho)\times(-\boldsymbol{a}_r)}{r^2}dS = \boldsymbol{a}_z\frac{-j\omega\sigma p_m t}{8\pi}\int_0^b\frac{\rho^3\,d\rho}{(d^2+\rho^2)^3}. \quad [\mathrm{A/m}] \quad (11)$$

Here $\boldsymbol{a}_\rho$ component would be zero when it is integrated with respect to φ due to its symmetry.

According to the similar assumption in calculation (6), it may be calculated as

$$\boldsymbol{H}(o) = \boldsymbol{a}_z H(o) = \boldsymbol{a}_z\frac{-j\omega\sigma p_m t}{32\pi}\frac{b^4}{d^2(d^2+b^2)^2}. \quad [\mathrm{A/m}] \quad (12)$$

Therefore magnetic flux density at 0 is

$$\boldsymbol{B}(o) = \boldsymbol{a}_z\frac{-j\omega\mu_0\sigma p_m t}{32\pi}\frac{b^4}{d^2(d^2+b^2)^2}. \quad [\mathrm{Wb/m^2}] \quad (13)$$

Then the electromotive force V'[V] induced along the coil due to Faraday's law of induction:

$$\begin{aligned} V' &= -j\omega\pi a^2 n B(o) \\ &= -\frac{\pi^3}{8}f^2\mu_0^2\sigma a^4 n^2 It\frac{b^4}{d^2(d^2+b^2)^2}. \quad [\mathrm{V}] \end{aligned} \quad (14)$$

2. Derivation of input impedance of the coil loaded by the semiconductor wafer

Fig. 2 shows the equivalent circuit of the coil received the electromotive force V'[V] from the semiconductor circular wafer. The equation

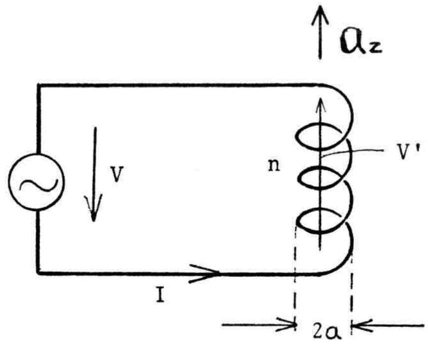

Fig. 2. A small coil is equivalent to a magnetic dipole with magnetic dipole moment:
$\boldsymbol{p}_m = \boldsymbol{a}_z \mu_0 \pi a^2\, In$ [Wb m]

$$j\omega LI = V + V'$$
$$= V - \frac{\pi^3}{8} f^2 \mu_0^2 \sigma a^4 n^2 It \frac{b^4}{d^2(d^2+b^2)^2}. \tag{15}$$

will hold for this equivalent circuit. Therefore input impedance of this coil will be given as

$$Z = \frac{V}{I} = j\omega L + \frac{\pi^3}{8} f^2 \mu_0^2 \sigma a^4 n^2 t \frac{b^4}{d^2(d^2+b^2)^2}. \quad [\Omega] \tag{16}$$

3. Measurement method of the change of input impedance of the coil when the semiconductor circular wafer approaching to the coil

Fig. 3 shows the diagram of the high frequency Wheatston brige for measuring the change of input impedance of the coil when the semiconductor circular wafer approaches to

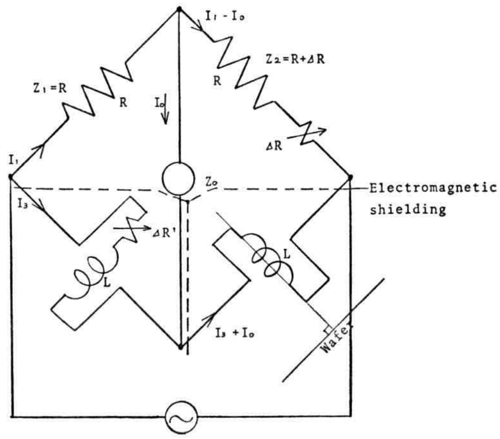

Fig. 3. High frequency Wheatston brige.

the coil. $Z_1=R$ is a non-reactive resistance, $Z_2=R+\Delta R$ is also non-reactive resistance including adjustable one ΔR. $Z_3=j\omega L+\Delta R'$ consists of a coil with inductance $L[H]$ and adjustable non-reactive resistance $\Delta R'$, $Z_4=j\omega L+\frac{\pi^3}{8}f^2\mu_0^2\sigma a^4 n^2 t\frac{b^4}{d^2(d^2+b^2)^2}$ is the impedance of the measuring coil which consists of a coil with inductance L[H] and changing part $\frac{\pi^3}{8}f^2\mu_0^2\sigma a^4 n^2 t\frac{b^4}{d^2({}^2d+b^2)^2}$ due to the wafer, Z_0 is the impedance of indicator.

At first, we remove the wafer and put $\Delta R'=0$, and then balance the Wheatston brige by adjusting ΔR. Secondly, we set the wafer and restore the balance by adjusting $\Delta R'$, then we have the relation:

$$\Delta R'=\frac{\pi^3}{8}f^2\mu_0^2\sigma a^4 n^2 t\frac{b^4}{d^2(d^2+b^2)^2}\,. \tag{17}$$

According to Kirchhoff's laws, we may calculate the current I_0 through the balance arm Z_0,

$$I_0=\frac{V_s(Z_1Z_4-Z_2Z_3)}{-Z_0(Z_1+Z_2)(Z_3+Z_4)\left\{1+\frac{1}{Z_0}\left(\frac{Z_1Z_2}{Z_1+Z_2}+\frac{Z_3Z_4}{Z_3+Z_4}\right)\right\}}\,. \quad [\mathrm{A}] \tag{18}$$

If we assume $Z_0 \gg Z_1, Z_2, Z_3, Z_4$, we would have

$$I_0 \doteqdot \frac{-V_s(Z_1Z_4-Z_2Z_3)}{Z_0(Z_1+Z_2)(Z_3+Z_4)}\,. \quad [\mathrm{A}] \tag{19}$$

when $Z_1=Z_2=R,\ \ Z_3=j\omega L,\ \ Z_4=j\omega L+\frac{\pi^3}{8}f^2\mu_0^2\sigma a^4 n^2 t\frac{b^4}{d^2(d^2+b^2)^2}$, we will obtain

$$I_0=j\frac{V_s}{Z_0}\frac{\pi^2}{64L}f\mu_0^2\sigma a^4 n^2 t\frac{b^4}{d^2(d^2+b^2)^2}\,. \quad [\mathrm{A}] \tag{20}$$

References

1) I. Murakami and M. Goto, "Theoretical Considerations on a Contactless Measuring Method of Electric Conductivity of Semiconductor Circular Wafer"; The Transactions of the IECE of Japan, vol. E65, no. 8 August 1982
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2) Ichiro Murakami, "Theory of Electicty and Magnetism", Maruzen, *p*. 155 (1978)
3) ditto, *p*. 124
4) Correct verification is given in reference 1).