

Earth-return Mutual Impedance between Overhead Power Line and Underground Communication Line

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Abstract

An analytical expression is given by Carson and Pollaczek in an integral form for the earth-return mutual impedance between overhead power line and underground communication line. In the forgoing paper,⁴⁾ the authors developed this integral form to an infinite series applying Watson's asymptotic expansion theorem for the case in which the two lines are separated by a large distance. In this paper is derived an analytical convergent series from the integral form referred to above, the series holding for an arbitrary separation of the two lines. In addition, it is confirmed that an approximate value of the mutual impedance can be obtained for practical purposes from a numerical computation of the above-mentioned convergent series.

1 Introduction

It is widely known that the earth-return mutual impedance between a pair of overhead lines was given in an integral form by Carson-Pollaczek's equations and can be calculated by use of the equations.^{1),2)} In addition, the earth-return mutual impedance between overhead power line and underground communication line is given by Carson-Pollaczek in an integral form.^{1),2)} L.M. Wadepholl and D.J. Wilcox developed this form to an infinite series.³⁾ However, the algorithm from which this infinite series is derived is not clearly presented in an explicit form, and an approximate numerical evaluation of this series seems not to have been developed, as yet. The authors therefore attempted to develop a method meeting practical purposes by extending the infinite series for cases treated by Carson of two overhead lines. The numerical value of the ensuing infinite series is compared with the one obtained from a numerical integration by applying Simpson's method. This comparison showed a satisfactory result.

2 Integral form

The earth-return mutual impedance between the two lines as shown in Fig. 1 is given by the following definite integral

$$Z_m = 4\omega(P + jQ) \times 10^{-4} = 4\omega \times 10^{-4} \int_0^\infty (\sqrt{\mu^2 + j} - \mu) \cos x' \mu e^{-(h' \mu + g' \sqrt{\mu^2 + j})} d\mu \quad (1)$$

where g' , h' and x' stand for the physical parameters (Fig. 1), P and Q representing a real function, respectively. r and θ are defined as follows,

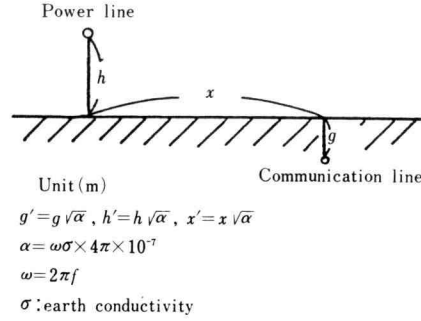


Fig. 1 Geometric configuration of two lines

$$r = \sqrt{x'^2 + (h' + g')^2}, \quad \theta = \tan^{-1}(x' / (h' + g'))$$

3 Infinite series form

Since $\cos x'\mu = (e^{jx'\mu} + e^{-jx'\mu})/2$, eqn. 1 leads to the following equation

$$P + jQ = \frac{1}{2}(M_1 + M_2) \quad (2)$$

where

$$M_1 = \int_0^\infty (\sqrt{\mu^2 + j} - \mu) e^{jx'\mu} e^{-(h'\mu + g'\sqrt{\mu^2 + j})} d\mu \quad (3)$$

$$M_2 = \int_0^\infty (\sqrt{\mu^2 + j} - \mu) e^{-jx'\mu} e^{-(h'\mu + g'\sqrt{\mu^2 + j})} d\mu \quad (4)$$

Let $z = h' + jx'$, $\sqrt{j}tr = z\mu + g'\sqrt{\mu^2 + j}$ and $\mu = 0$, then $t = g'/r$. Therefore, let $z = h' + jx'$ and $\sqrt{j}tr = z\mu + g'\sqrt{\mu^2 + j}$, then $\mu = \frac{\sqrt{j}t}{z^2 - g'^2}(zt - g'\sqrt{t^2 + (z^2 - g'^2)/r^2})$ and

$$M_1 = \frac{1}{z - g'} \left\{ \frac{jr^2}{z - g'} \left(\int_0^\infty \sqrt{t^2 + \beta} e^{-\sqrt{j}tr} dt - \frac{g'}{\sqrt{j}r^2} e^{-\sqrt{j}g'} - \frac{1}{jr^2} e^{-\sqrt{j}g'} \right) - \int_0^{g'/r} \sqrt{t^2 + \beta} e^{-\sqrt{j}tr} dt \right\} - jg' \int_0^\infty \frac{e^{-\sqrt{j}tr}}{\sqrt{t^2 + \beta}} dt + jg' \int_0^{g'/r} \frac{e^{-\sqrt{j}tr}}{\sqrt{t^2 + \beta}} dt \Bigg\},$$

where

$$\beta = (z^2 - g'^2)/r^2 \quad (5)$$

First,

$$\int_0^\infty \sqrt{\nu^2 + \gamma^2} e^{-\beta\nu} d\nu = \frac{\gamma}{\beta} \left\{ \frac{\pi}{2} H_1(\beta\gamma) - \frac{\pi}{2} N_1(\beta\gamma) \right\}^{(5)} \quad (6)$$

$$\int_0^\infty \frac{e^{-\beta\nu}}{\sqrt{\nu^2 + \gamma^2}} d\nu = \frac{\pi}{2} \left\{ H_0(\beta\gamma) - N_0(\beta\gamma) \right\} \quad (7)$$

where

$$H_\nu(z) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi}} \left(\frac{z}{2}\right)^\nu \frac{(-1)^n z^{2n+1}}{(2n+1)! 2^n \Gamma\left(\nu + n + \frac{3}{2}\right)} \quad (8)$$

$$\Gamma\left(n + \frac{1}{2}\right) = (2n-1)!! / 2^n \times \sqrt{\pi}$$

$$\frac{\pi}{2} N_0(z) = A_0 - B_0, \quad \pi N_1(z) = -C_1 + 2A_1 - (B_1 - D_1)^{6)} \quad (9)$$

$$A_0 = J_0(z) \ln \frac{\gamma z}{2} \quad (\gamma = 1.7811)$$

$$B_0 = -\left(\frac{z}{2}\right)^2 + \frac{1+\frac{1}{2}}{2!^2} \left(\frac{z}{2}\right)^4 - \frac{1+\frac{1}{2}+\frac{1}{3}}{3!^2} \left(\frac{z}{2}\right)^6 + \dots$$

$$J_0(z) = \frac{\left(\frac{1}{2}z\right)^0}{0!^2} - \frac{\left(\frac{1}{2}z\right)^2}{1!^2} + \frac{\left(\frac{1}{2}z\right)^4}{2!^2} - \dots$$

$$C_1 = 2/z, \quad A_1 = J_1(z) \ln \frac{\gamma z}{2},$$

$$B_1 = J_1(z) - \frac{1}{1!2!} \left(\frac{z}{2}\right)^3 + \frac{1}{2!3!} \left(\frac{z}{2}\right)^5 - \dots,$$

$$D_1 = \left[\frac{z}{2} - J_1(z)\right] - \frac{1}{2!3!} \left(\frac{z}{2}\right)^5 + \frac{1}{3!4!} \left(\frac{z}{2}\right)^7 - \dots,$$

$$J_1(z) = \frac{\left(\frac{1}{2}z\right)}{0!1!} - \frac{\left(\frac{1}{2}z\right)^3}{1!2!} + \frac{\left(\frac{1}{2}z\right)^5}{2!3!} - \dots$$

By using eqns. 8 and 9, the right-hand-sides of eqns. 6 and 7 are developed to an infinite series.

Secondly,

$$\int_0^{g'/r} \sqrt{t^2 + \beta} e^{-\sqrt{\beta} t r} dt = \sum_{n=0}^{\infty} \frac{(-\sqrt{\beta} r)^n}{n!} \int_0^{g'/r} \sqrt{t^2 + \beta} t^n dt$$

and

$$\int_0^{g'/r} \frac{e^{-\sqrt{\beta} t r}}{\sqrt{t^2 + \beta}} dt = \sum_{n=0}^{\infty} \frac{(-\sqrt{\beta} r)^n}{n!} \int_0^{g'/r} \frac{t^n}{\sqrt{t^2 + \beta}} dt \quad (10)$$

$$\text{Let } \sqrt{t^2 + \beta} = t + s,$$

then

$$\begin{aligned} \int_0^{g'/r} \sqrt{t^2 + \beta} t^n dt &= -\frac{(-1)^n}{2^{n+2}} \left[\sum_{j=0}^n {}_n C_j \frac{s^{2j+2-n}}{2j+2-n} (-\beta)^{n-i} + 2\beta \sum_{i=0}^n {}_n C_i \frac{s^{2i-n}}{2i-n} (-\beta)^{n-i} \right. \\ &\quad \left. + \beta^2 \sum_{i=0}^n {}_n C_i \frac{s^{2i-n-2}}{2i-n-2} (-\beta)^{n-i} \right]_{s=\sqrt{\beta}}^{s=\sqrt{g'^2/r^2 + \beta} - g'/r} \end{aligned}$$

and

$$\int_0^{g'/r} \frac{t^n}{\sqrt{t^2 + \beta}} dt = \frac{(-1)^{n+1}}{2^n} \sum_{i=0}^n (-\beta)^{n-i} {}_n C_i \left[\frac{s^{2i-n}}{2i-n} \right]_{s=\sqrt{\beta}}^{s=\sqrt{g'^2/r^2 + \beta} - g'/r} \quad (11)$$

where $\frac{s^0}{0}$ is defined by $\log_e s$.

By using eqns. 6, 7, 10 and 11, the right-hand-side of eqn. 5 is developed to the following infinite series

$$\begin{aligned}
 M_1 = & \frac{1}{z-g'} \left\{ \frac{j r^2}{z-g'} \left(\frac{\sqrt{\beta}}{\sqrt{j} r} \left(\frac{\pi}{2} H_1(\sqrt{\beta} \sqrt{j} r) - \frac{\pi}{2} N_1(\sqrt{\beta} \sqrt{j} r) \right) - \frac{g'}{\sqrt{j} r^2} e^{-\sqrt{j} g'} - \frac{1}{j r^2} e^{-\sqrt{j} g'} \right. \right. \\
 & + \sum_{n=0}^{\infty} \frac{(-\sqrt{j} r)^n}{n!} \frac{(-1)^n}{2^{n+2}} \left[\sum_{i=0}^n n C_i \frac{y^{2i+2-n}}{2i+2-n} (-\beta)^{n-i} + 2\beta \sum_{i=0}^n n C_i \frac{y^{2i-n}}{2i-n} (-\beta)^{n-i} \right. \\
 & + \beta^2 \sum_{i=0}^n n C_i \frac{y^{2i-n-2}}{2i-n-2} (-\beta)^{n-i} \left. \right]_{y=\sqrt{g'^2/r^2+\beta}-g'/r} \left. - j g' \frac{\pi}{2} (H_0(\sqrt{\beta} \sqrt{j} r) - N_0(\sqrt{\beta} \sqrt{j} r)) \right. \\
 & \left. + j g' \sum_{n=0}^{\infty} \frac{(-\sqrt{j} r)^n}{n!} \frac{(-1)^{n+1}}{2^n} \sum_{i=0}^n (-\beta)^{n-i} n C_i \left[\frac{y^{2i-n}}{2i-n} \right]_{y=\sqrt{\beta}} \right\} \quad (12)
 \end{aligned}$$

where $\frac{y^0}{0}$ is defined by $\log_e y$.

M_2 is likewise obtained from eqn. 12 by replacing z and β with \tilde{z} and $\tilde{\beta}$, respectively-where $\tilde{z} = h' - jx'$, $\tilde{\beta} = (\tilde{z}^2 - g'^2)/r^2$.

4 Examination of eqns. 2 and 12

Let $r=6$ and $0 \leq g' \leq 1$ in eqn. 2, then Z_m nearly corresponds to the value which was obtained in the forgoing paper (Fig. 2). In addition, let $g'=0$ in eqn. 12, then eqn. 2 leads to the following equation

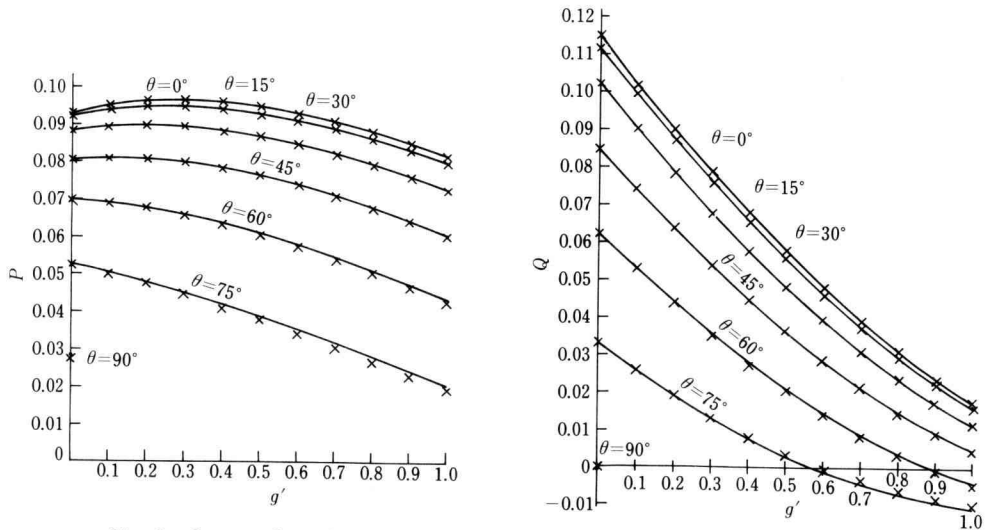


Fig. 2 Computed results of $P, Q(r=6)$

× : Computed from the asymptotic expansion (Reference 4, p. 175-p. 176)

$$\begin{aligned}
 Z_m = & \frac{1}{2} \times 4\omega \times 10^{-4} \left[\frac{\sqrt{j}}{z} \left(\frac{\pi}{2} H_1(z\sqrt{j}) - \frac{\pi}{2} N_1(z\sqrt{j}) \right) - \frac{1}{jzr^2} + \frac{\sqrt{j}}{\tilde{z}} \left(\frac{\pi}{2} H_1(\tilde{z}\sqrt{j}) \right. \right. \\
 & \left. \left. - \frac{\pi}{2} N_1(\tilde{z}\sqrt{j}) \right) - \frac{1}{j\tilde{z}r^2} \right] \quad (13)
 \end{aligned}$$

Table 1 Comparison of the approximate and the numerically integrated values ($\backslash : h' < 0$)

	r	θ	g'	Numerical ^(**) integration	Approximate ^(*) value	Relative error (%)
P	1	0	0	0.256338	0.256365	0.01
		0	1	0.323235	0.323190	0.02
		30	0	0.260328	0.260354	0.01
		30	1	\backslash	\backslash	\backslash
		60	0	0.274239	0.274262	0.01
		60	1	\backslash	\backslash	\backslash
		90	0	0.305418	0.305379	0.01
		90	1	\backslash	\backslash	\backslash
	6	0	0	0.093550	0.093505	0.05
		0	1	0.081760	0.081728	0.04
		30	0	0.088105	0.088003	0.12
		30	1	0.072614	0.072608	0.08
		60	0	0.069473	0.069518	0.06
		60	1	0.043627	0.043617	0.02
		90	0	0.028713	0.028718	0.02
		90	1	\backslash	\backslash	\backslash
Q	1	0	0	0.505171	0.505232	0.01
		0	1	0.216019	0.216080	0.03
		30	0	0.488971	0.489030	0.01
		30	1	\backslash	\backslash	\backslash
		60	0	0.439388	0.439136	0.06
		60	1	\backslash	\backslash	\backslash
		90	0	0.349098	0.352362	0.93
		90	1	\backslash	\backslash	\backslash
	6	0	0	0.114911	0.114901	0.01
		0	1	0.017404	0.017405	0.01
		30	0	0.101813	0.101766	0.05
		30	1	0.011366	0.011328	0.33
		60	0	0.062492	0.062445	0.08
		60	1	-0.003669	-0.003637	0.82
		90	0	-0.000832	-0.000847	1.80
		90	1	\backslash	\backslash	\backslash

(*) computed from eqns. 2 and 12

(**) computed by Simpson's method

This result Z_m corresponds to the series derived by Carson of the mutual impedance between a pair of overhead lines. It may be interesting to note that this series presented by Carson is obtained as a special case of eqns. 2 and 12. It is also confirmed that the results of eqns. 2 and 12 remain within a difference of one percent (Table 1) in comparison with the result from a numerical integration by Simpson's method except the case of $r=6$, $\theta=90^\circ$, $g'=0$. However, since the magnitude itself of the value for this case is very small, the relative

error turns out rather large.

5 Conclusion

In this paper, the following items are confirmed ;

(1) The earth-return mutual impedance between overhead power line and underground communication lines can be obtained from an infinite series expansion of the definite integral which was derived by Carson and Pollaczek.

(2) The approximate value of the impedance can be obtained for practical purposes by computing the infinite series expansion given by eqns. 2 and 12.

(3) In case of $r=6$, the result Z_m , which was obtained in the forgoing paper, nearly corresponds to the value obtained in this paper.

References

- 1) CARSON, J.R. : B.S.T.J. vol. 5 p. 539 (1926-10)
- 2) POLLACZEK, F. : E.N.T. vol. 3 p. 339 (1926-9)
- 3) WEDEPOHL, L.M. and WILCOX, D.J. : Transient analysis of underground power-transmission systems (System-model and wave-propagation characteristics), PROC. IEE, vol. 120, No. 2, February 1973, p. 253
- 4) TAKAHASHI, S., OHYA, S. and MORI, T. : Research Reports of Ikutoku Tech. Univ. B-7 p. 173 (1982)
- 5) MORIGUCHI, UDAGAWA and HITOTSUMATSU : Table 3 of mathematical formulas ; Iwanami, p. 182, p. 228, p. 227
- 6) JAHNKE and EMODE : Table of Functions ; Dover, p. 130-p. 132