

Quantum Black Hole in Two Dimensions (1)

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Abstract

Callen Giddings, Harvey and Strominger (CGHS) have investigated an interesting two dimensional ($2D$) dilatonic gravity which shows the black hole evaporation in the semi-classical approximation. Their work includes the $2D$ black hole solution in critical string theory researched by Witten. Generally the black holes evaporate completely without a singularity in terms of Hawking radiation and its back reaction of the metric. It can be, however, pointed out that in the case of $2D$ dilatonic black hole the semiclassical equations give a new singularity hidden inside a black hole.

1. Introduction

The research of $2D$ dilatonic gravity has been one of the most exciting, remarkable studies in the recent elementary particle physics^{[1]–[6]}. The aim of the research of $2D$ dilatonic gravity model is that we will make another step close to the complete construction of the quantum theory of gravity. Actually, it was difficult for us to construct four-dimensional quantum gravity. Now we have recently constructed a toy model for $2D$ quantum gravity^{[7]–[11]}. The model is within a framework of the dimensional reduction from full $4D$ to $2D$. After the complete construction of $2D$ theory, we will be able to obtain the perspective for $4D$ theory.

Callen, Giddings, Harvey and Strominger (CGHS) have investigated an interesting two-dimensional dilatonic gravity which shows the black hole evaporation in the semi-classical approximation^[1]. Their model includes the **$2D$ black hole** solution in critical string theory researched by Witten^[12]. In this CGHS model, the gravity couples with a dilaton and conformal matter fields. They found classical exact solutions, including the solutions describing the formation of a black hole by collapsing matter.

Generally in the CGHS model the black hole evaporate completely, in terms of Hawking radiation and its back reaction of the metric, without a singularity. It has been, however, pointed out that in the case of $2D$ dilatonic black hole the semiclassical equations give a new singularity hidden inside a black hole^{[2]–[5]}. Consequently, following the quantum effects, the quantum version of $2D$ dilatonic gravity model^[6] is presented by means of the procedure of $2D$ quantum gravity of Distler and Kawai^{[7]–[9]}. Recently, the formation of worm hole in $2D$ space-time has been also argued, using the procedure by Oda and Nojiri with $SL(2, R)/U(1)$ gauged WZW model^[20].

In this paper the authors review black hole solutions in string theory and mainly the

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progress of CGHS dilatonic black hole in these few years.

2. String Theory and 2D Black Hole

Much of the recent progress in string theory has been based on the insight that consistent string theories can be constructed in dimensions much lower than the critical dimension. The matrix model approach has proved to be successful in studying toy models of string theory. These non-critical string theories are conventionally formulated on the world-sheet as a $c=1$ conformal field theory (CFT) coupled to $2D$ gravity.

In addition, it also turns out that string theories has a rich variety of solutions describing extended objects surrounded by event horizons. These theories also include **black string** solutions in ten dimensions characterized by three parameters, the mass and axion charge per unit length, and asymptotic value of the dilaton. These solutions are obtained by solving the lowenergy string equations of motion^[13–15]. It has been shown that three-dimensional black strings are most naturally described in terms of the string metric appearing in the sigma model.

Witten has recently shown that $SL(2, R)/U(1)$ gauged Wess-Zumino-Witten (WZW) model yields **2D black hole**^[12], which raises the problem of the similar way in finding exact conformal field theories (CFT) corresponding to higher dimensional black holes or extended black holes.

In this section we argue that an exact CFT describing black holes in $2D$ space-time is found as $SL(2, R)/U(1)$ gauged WZW model. Our purpose in this section is that we construct CFT describing a black hole in $2D$ target space-time. CFT governing the black hole is presumably soluble owing to the extended chiral algebra of $SL(2, R)/U(1)$. Black hole solutions of field theories in lowenergy limit, derived from string theory in four dimensions, have been analyzed. We confine now ourselves the bosonic string theory. This section cannot help being argued as historical outlines because of so much of progress in $2D$ string theories.

First we review a critical string theory in $2D$ target spacetime^[16]. The usual continuum world-sheet action in $2D$ spacetime with coordinates $X^i=(\sigma, x)$ is

$$S = \frac{1}{8\pi} \int d^2\xi \sqrt{-h} \{ h^{ab} G_{ij}(X) \partial_a X^i \partial_b X^j - 2R^{(2)}\phi(X) + 2T(X) \}, \quad (1)$$

where σ and x are space-like and time-like coordinates in Minkowski metric respectively. The local fields $G_{ij}(X)$, $\phi(X)$ and $T(X)$ are corresponding respectively to space-time metric, dilaton field and tachyon field. $R^{(2)}$ is the curvature of the world-sheet metric h^{ab} . The above action describes CFT when the following conditions are satisfied,

$$\begin{aligned} G_{ij} &= \eta_{ij}, \\ \phi &= -\sqrt{2}\sigma, \\ T(X) &= 0. \end{aligned} \quad (2)$$

We consider the energy-momentum tensor,

$$T(z) = \frac{1}{2} \{ (\partial x)^2 - (\partial \sigma)^2 \} - \sqrt{2} \partial^2 \sigma. \quad (3)$$

This tensor satisfies the usual operator product expansion with $c=26$, central charge. We call this solution the linear dilaton vacuum.

Next let us now argue $2D$ black hole solution in string theory. The linear dilaton vacuum as a solution to the fixed point equation is a special case of the following black hole solution,

$$e^{-2\phi} = e^{-w} = uv - C \quad (C = \text{const.}), \quad T=0, \quad (4)$$

$$u = e^{\frac{1}{\sqrt{2}}(\sigma+x)},$$

$$v = -e^{\frac{1}{\sqrt{2}}(\sigma-x)}, \quad (5)$$

in the conformal gauge for $2D$ target space-time metric,

$$G_{ij} = e^w \eta_{ij}, \quad (6)$$

where u and v are light-like coordinates and w is called the Liouville mode or the conformal mode. The solution reduces to the linear dilaton vacuum in the limit $\sigma \rightarrow \infty$ or $C \rightarrow 0$. The exact CFT describing black hole solution is given by Witten^[12]. In this case the theory is $SL(2, R)/U(1)$ WZW model with level $k=9/4$ in the central charge of corresponding current algebra. The action is obtained by the gauge-invariant generalization of the WZW action and is given by

$$\begin{aligned} S_{WZW} = & -\frac{k}{8\pi} \int_{RS} \text{Tr}[(g^{-1} \partial_+ g)(g^{-1} \partial_- g)] - ik T_{WZ} \\ & - \frac{k}{4\pi} \int \text{Tr}(A_- g^{-1} \partial_+ g + A_+ g^{-1} \partial_- g - 2A_+ A_- + A_+ g A_- g^{-1}). \end{aligned} \quad (7)$$

where $g: RS \rightarrow SL(2, R)$ is the field variable of the model, RS is a Riemann surface, Tr is the trace in $2D$ representation of $SL(2, R)$, k is a positive real number, T_{WZ} is the Wess-Zumino term,

$$T_{WZ}(g) = \frac{1}{12\pi} \int_B \text{Tr} g^{-1} dg \wedge g^{-1} dg \wedge g^{-1} dg, \quad (8)$$

and A_i is an Abelian gauge field,

$$\delta A_i = -\partial_i \epsilon \quad (9)$$

B is any three-manifold with boundary RS and we use an extension of g to a map from B to $SL(2, R)$. The central charge of $SL(2, R)/U(1)$ gauged WZW model with level k is

$$c = \frac{3k}{k-2} - 1 = 2 + \frac{6}{k-2}. \quad (10)$$

The sigma model describing the black hole is weakly coupled in the world sheet sense when $k \rightarrow \infty$. We adjoin additional matter of central charge near 24 to obtain a bosonic string background. The black hole has $c=26$ for $k=9/4$ from (10) and can be considered as bosonic

string theory.

We now consider the status of the black hole in string theory. It has been recently shown that the graviton-dilaton field equations,

$$\begin{aligned} ds^2 &= -\frac{dudv}{(1-uv)}, \\ \phi(u, v) &= \ln(1-uv), \end{aligned} \quad (11)$$

mean the essence of the black hole and give CFT where $\phi(u, v)$ is the dilaton field. The analytic continuation past the coordinate singularity can be found by imitating the similar procedure for the Schwarzschild solution. In this case CFT with $2D$ target space is parametrized by two variables u and v ,

$$\begin{aligned} 2v &= e^{r'+t}, \quad 2u = -e^{r'-t}, \\ r' &= r + \ln(1 - e^{-2r}), \end{aligned} \quad (12)$$

where $2D$ target space metric of this theory is

$$ds^2 = dr^2 - \tanh^2 r \, dt^2 \quad (13)$$

According to the space-time diagram of $2D$ black hole in (u, v) -coordinates, the field equations (11) show an event horizon at $uv=0$ as well as a curvature singularity at $uv=1$ just as the Schwarzschild black hole in Kruskal-coordinates. As a result, a world-sheet action, in terms of Liouville theory, is given by

$$S = \int d^2\sigma \left\{ \left(\frac{\partial_i u \partial^i v}{1-uv} \right) + R^{(2)} \phi(u, v) \right\}, \quad (14)$$

furthermore, the gauged WZW action turns out to be

$$S_{WZW} = -\frac{k}{4\pi} \int d^2x \sqrt{-h} (h^{ij} \partial_i u \partial_j v / (1-uv)). \quad (15)$$

The above action exhibits directly the characteristic features of a black hole geometry. In the lowest order world-sheet perturbation theory, the graviton-dilaton system in string theory can be converted into a space-time effective action

$$S_{eff} = \int d^2x \sqrt{-g} e^{\phi} \{ R + (\nabla \phi)^2 + 8 \}. \quad (16)$$

Consequently, Witten showed, with the aid of CFT techniques, how to construct an exact solution to string theory, which reduces to the black hole in the low-energy limit. In addition to the above connection between string theory and $2D$ black hole, especially the effective action (16) is the starting point for CGHS dilaton gravity argued in the next section.

3. CGHS Dilatonic Black Hole

Callan, Giddings, Harvey and Strominger (CGHS) have studied $2D$ dilatonic black hole^[1]. Their model (CGHS model) is a renormalizable theory of $2D$ gravity coupled to dilaton field and conformal matter fields. Furthermore the black hole solution of CGHS model in the

absence of matter appears as a low-energy approximation to an exact solution of Witten's CFT describing 2D black hole argued in the previous section^[12]. The CGHS model has exact classical solutions where black holes are formed in gravitational collapse. In this section we consider semiclassical CGHS theory.

We begin with the following action in 2D space-time,

$$S_{CGHS} = \frac{1}{2\pi} \int d^2x \sqrt{-g} \{ e^{-2\phi} [R + 4(\nabla\phi)^2 - 4\lambda^2] - \frac{1}{2} \sum_{i=1}^N (\nabla f_i)^2 \}. \quad (17)$$

Here g , ϕ and f_i are the metric, dilaton and a set of N massless minimally coupled scalar fields, respectively, and λ^2 is a cosmological constant. It is very closely related to 2D target space-time action of $c=1$ non-critical string theory. It has been considered that the process of formation and evaporation of black holes is not governed by the usual laws of quantum mechanics. We have intended to obtain an effective action describing the back reaction of matter and Hawking radiation on geometry. In 2D space-time, CGHS model actually allows us to describe Hawking radiation and its back reaction on the metric.

The CGHS theory is analyzed in the conformal gauge,

$$g_{+-} = -\frac{1}{2} e^{2w}, \quad g_{++} = g_{--} = 0 \quad (18)$$

where we use the light-cone coordinates,

$$x^+ = x^0 + x^1, \quad x^- = x^0 - x^1. \quad (19)$$

We then have $R = 8e^{-2\phi} \partial_+ \partial_- w$. The full effective action in conformal gauge is

$$S_{\text{eff}} = \int d^2x e^{-2\phi} (-2\partial_+ \partial_- w + 4\partial_+ \phi \partial_- \phi - \lambda^2 e^{2w}) + k \partial_+ w \partial_- w - \frac{1}{2} \partial_+ f_i \partial_- f_i, \quad (20)$$

where $k = N\hbar/12$. Therefore the dilaton and matter field equations of motion are given by

$$\begin{aligned} -4\partial_+ \partial_- \phi + 4\partial_+ \phi \partial_- \phi + 2\partial_+ \partial_- w + \lambda^2 e^{2w} &= 0, \\ \partial_+ \partial_- f_i &= 0, \end{aligned} \quad (21)$$

where we use $\delta S / \delta \phi = 0$ and $\delta S / \delta f_i = 0$. The metric equation reduces to

$$T_{++} = e^{-2\phi} (4\partial_+ w \partial_+ - 2\partial_+^2 \phi) + \frac{1}{2} \partial_+ f_i \partial_+ f_i = 0. \quad (22)$$

where we use $T_{ij} = \left(\frac{2\pi}{\sqrt{-g}} \right) \delta S / \delta g^{ij} = 0$. The above equation is called the constraint equation.

We now consider exact solutions of the above equations (21). The simplest one is the vacuum solution,

$$f_i = 0, \quad e^{-2\phi} = e^{-2w} = -\lambda^2 x^+ x^-. \quad (23)$$

This non-trivial solution means just the linear dilaton vacuum of string theory argued in the previous section. A change of coordinates,

$$x^+ = +\exp(+u^+), \quad x^- = -\exp(-u^-), \quad (24)$$

gives a flat metric and a linear dilaton field,

$$w=0, \quad \phi = -\frac{1}{2} \lambda (u^+ - u^-) = -\lambda u^1, \quad (25)$$

where $u^+ = u^0 + u^1$ and $u^- = u^0 - u^1$. The general solution of the equations is obtained as the following static solution,

$$f_i=0, \quad e^{-2\phi} = \frac{M}{\lambda} - \lambda^2 x^+ x^- = e^{-2w}, \quad (26)$$

where M is an integration constant and actually corresponds to the black hole mass. A static black hole is described by the above equation. For $M=0$, the metric is flat and the dilaton field is linear. For $M>0$, the equation corresponds to $2D$ black hole solution (see Fig. 1). Let us calculate the curvature ;

$$R = 8e^{-2w} \partial_+ \partial_- w = 4M\lambda / (\frac{M}{\lambda} - \lambda^2 x^+ x^-), \quad (27)$$

which is divergent at $x^+ x^- = M/\lambda^3$. This solution has the same qualitative features as the (r, t) plane of Schwarzschild black hole.

We now discuss Hawking radiation and the back reaction.

First we consider the formation of a black hole from the conformal matter. The f^i are a set of N massless matter fields, The matter propagates along null geodesics. We consider the f^i as the following function,

$$f_i = f_{i+}(x^+) + f_{i-}(x^-). \quad (28)$$

We introduce the f^i matter shock wave (f -wave), traveling in the x^- direction with magnitude a , described by the stress tensor,

$$\frac{1}{2} \partial_+ f_i \partial_+ f_i = a \delta(x^+ - x^+_0). \quad (29)$$

The only modification in the equations and constraints due to the matter fields in this case is including in the g^{++} constraint which turns out

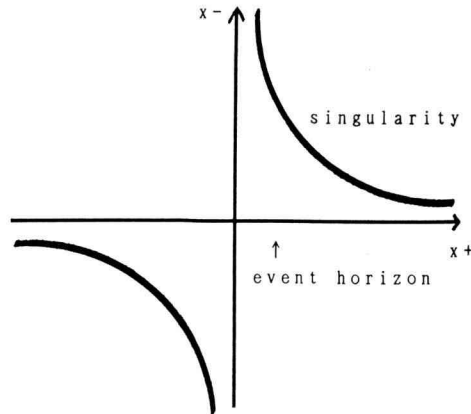


Fig.1 The Kruskal diagram for black hole

$$e^{-2\phi}(4\partial_+ w \partial_+ \phi - 2\partial_+^2 \phi) = -\frac{1}{2} \cdot \partial_+ f_i \partial_+ f_i. \quad (30)$$

For $x^+ < x_0^+$, this is simply the linear dilaton vacuum, while, for $x^+ > x_0^+$, we know that the solution must become the general form (26). We match the discontinuity across x_0^+ and obtain the solution,

$$\begin{aligned} e^{-2\phi} &= e^{-2w} \\ &= -a(x^+ - x_0^+) \Theta(x^+ - X_0^+) - \lambda^2 x^+ x^- \\ &= -\frac{M}{\lambda x_0^+} \cdot (x^+ - x_0^+) - \lambda^2 x^+ x^-. \end{aligned} \quad (31)$$

For $x > x_0^+$, this is identical to a black hole of mass $ax_0^+ \lambda$ after shifting x^- by a/λ^2 (see Fig. 2). The f -wave produces a black hole and the weak-field perturbation breaks down since the weak-field expansion parameter is proportional to e^ϕ . The above equation (31) means that the parameter is arbitrarily large close to infinity (+, -) or to the singularity and that the weak-field expansion diverges in this region.

Secondly we analyze the quantum effects of matter fields in the fixed classical background of a black hole formed by collapsing matter. In $2D$ space-time we have a beautiful relation between the trace anomaly and Hawking radiation. The trace anomaly is given as the expectation value of the trace of the energy-momentum tensor,

$$\langle T_i^i \rangle = \frac{c}{24} R. \quad (32)$$

In the conformal gauge, for N scalar fields ($c=1$) we can calculate T_{+-}^f . The result is

$$\langle T_{+-}^f \rangle = -\frac{N}{12} \partial_+ \partial_- w, \quad (33)$$

where $T = -4e^{-2w} T_{+-}$. Similarly we obtain

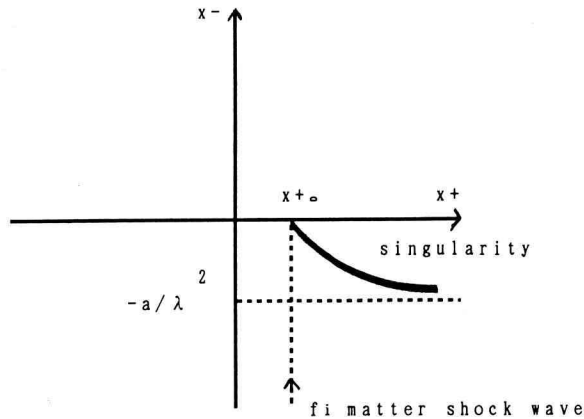


Fig. 2 The Kruskal diagram for formation of black hole by f_i matter shock wave

$$\begin{aligned}\langle T^f_{++} \rangle &= -\frac{N}{12} \{ \partial_+ w \partial_+ w - \partial_+^2 w + t_+(x_+) \}, \\ \langle T^f_{--} \rangle &= -\frac{N}{12} \{ \partial_- w \partial_- w - \partial_-^2 w + t_-(x_-) \},\end{aligned}\quad (34)$$

where we use the energy-momentum conservation, $T^{++}=2\partial_+w$ and $T^{--}=2\partial_-w$. We now calculate Hawking radiation from a physical black hole formed by collapse of an infalling f -wave. The coordinates (x^+, x^-) are changed into the following ones,

$$\begin{aligned}X^+ &= \frac{1}{\lambda} \ln(\lambda x^+), \\ X^- &= -\frac{1}{\lambda} \ln(-\lambda x^- - a/\lambda),\end{aligned}\quad (35)$$

where $\lambda x^+_0 = \exp(X^+_0)$. The function of integration t_+, t_- must be fixed by boundary conditions on T^f at infinity. The formula of w implies

$$\begin{aligned}t_+ &= 0, \\ t_- &= -\frac{\lambda^2}{4} \{ 1 - (1 + a \exp(\lambda X^-)/\lambda)^{-2} \}.\end{aligned}\quad (36)$$

By taking $X \rightarrow \infty$ we obtain the limiting value of the stress tensor,

$$\begin{aligned}\langle T^f_{+-} \rangle &\rightarrow 0, \quad \langle T^f_{--} \rangle \rightarrow 0, \\ \langle T^f_{--} \rangle &\rightarrow N\lambda^2/48 \cdot \left\{ 1 - \frac{1}{(1 + a \exp(\lambda X^-)/\lambda)^2} \right\}.\end{aligned}\quad (37)$$

As a result, the limiting value of T^f approaches the constant value $N\lambda/48$ in the far past of infinity, $X \rightarrow -\infty$. The above discussion means Hawking radiation.

Black holes can never radiate more energy than they possess. The back reaction of Hawking radiation on the geometry actually has been neglected. Therefore we should regard that the back reaction is ultimately important enough to terminate the radiation process when the black hole mass reaches zero. The back reaction is included by letting the quantum stress tensor (33) and (34) act as a source for the classical metric equations. The w equation of motion is given by

$$e^{-2\phi}(2\partial_+\partial_-\phi - 4\partial_+\phi\partial_-\phi - \lambda^2 e^{2w}) = 0. \quad (38)$$

Furthermore the modified version of the above equation is

$$e^{-2\phi}(2\partial_+\partial_-\phi - 4\partial_+\phi\partial_-\phi - \lambda^2 e^{2w}) = \frac{N}{12} \partial_+\partial_-\phi, \quad (39)$$

which can be derived from the non-local Polyakov action,

$$S_P = -\frac{N}{96} \int d^2x \sqrt{-g} R \square^{-1} R, \quad (40)$$

where $\square^{-1}R = -2w$ in the conformal gauge. By adding the action (40) to the classical action (17), we obtain the quantum effective action to be solved,

$$\begin{aligned}
S_N &= S + \left(-\frac{N}{96}\right) \int d^2x \sqrt{-g} R \square^{-1} R, \\
&= \frac{1}{\pi} \int d^2x [e^{-2\phi} (-2\partial_+ \partial_- w + 4\partial_+ \phi \partial_- \phi - \lambda^2 e^{2w}) \\
&\quad - \sum_{i=1}^N \frac{1}{2} \cdot \partial_+ f_i \partial_- f_i + \frac{N}{12} \partial_+ w \partial_- w].
\end{aligned} \tag{41}$$

The last term is called the Liouville term. It is induced by the N matter fields. The conformal gauge constraint equations are modified by the presence in a way which will shortly be made explicit. When we consider the strong coupling region, $g^2 = e^{2\phi}$, the dilaton gravity term is then very small, and the action governing w and f_i reduces to Liouville gravity coupled to the conformal matter,

$$S_N = \frac{1}{\pi} \int d^2x \left[-\frac{1}{2} \sum_{i=1}^N \partial_+ f_i \partial_- f_i + \frac{N}{12} \partial_+ w \partial_- w \right]. \tag{42}$$

The $+-$ constraint equation is

$$0 = T_{+-} = -\frac{N}{12} \partial_+ \partial_- w \propto R. \tag{43}$$

The T_{+-} constrain implies that space-time is FLAT. CGHS suggested that a black hole formed from f -wave would evaporate completely without a singularity, in terms of Hawking radiation and its back reaction of the metric. It has been, however, pointed out that in the case of 2D dilatonic black hole the semi-classical equations give a new singularity hidden inside a black hole^[2,3,5].

The geometry above f -wave can perturbatively computed in a Taylor expansion about f -wave. We find f -wave,

$$\partial_+ \phi(x^+_0, x^-) = \frac{1}{2x^+_0} \cdot \left(\frac{M/\lambda}{\sqrt{P(\phi(x^+_0, x^-))}} - 1 \right), \tag{44}$$

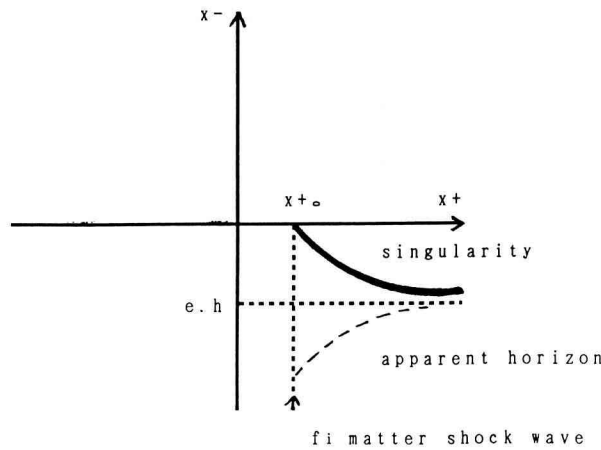


Fig. 3 The Kruskal diagram for large- N black hole formation/evaporation

where by continuity $\phi(x^+_0, x^-)$ is given by its vacuum value. We have features of this expression. $\partial_+\phi$ diverges when f -wave crosses the time-like line in the vacuum where $\phi = \phi_{cr} = 1/2 \ln(12/N)$. Before diverging it must cross zero at an earlier value of x^- . This point marks the beginning of an apparent horizon. Behind this horizon and above f -wave we have trapped region or an apparent black hole. The singularity at $\phi = \phi_{cr}$ is inside an apparent black hole (see Fig. 3).

Discussion

In the previous section we discussed $2D$ metric dilaton system coupled to matter fields as a simple models by the semi-classical theory (CGHS) of black holes. The full quantum theory of $2D$ dilaton gravity, however, is not yet solvable. In order to incorporate the quantum effects in lowest order CGHS model includes the contribution of the conformal anomaly coming from the conformally non invariant measure in the matter sector path integral. Full quantization of CGHS model was recently carried out^[6] by following the procedure of David, and of Distler, Hlousek and Kawai^[7-9]. The quantum analysis of this model was discussed using techniques of conformal field theory (CFT). Recently another quantum theory of $2D$ dilaton gravity is analyzed by Oda and Nojiri (ON)^[20]. The ON model is described by $SL(2, R)/U(1)$ gauged WZW model deformed by $(1, 1)$ operator. They have showed that the curvature singularity does not appear when central charge c_{matter} of the matter fields is given by $22 < c_{matter} < 24$. When $22 < c_{matter} < 24$, the matter shock waves create a kind of worm holes.

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