Lattice Quantum Gravity on Critical String

Hiroshi S. EGAWA* and Yoshiichi FUKUDA*

Abstract

In the lattice approach of two-dimensional quantum gravity, in order to obtain the partition function all possible dynamical triangulated surfaces are summed up where each triangle is a regular triangle with the same size. We propose a method to define the complex structure and separate the conformal mode on two-dimensional dynamically triangulated surfaces. The complex structure is well-defined at $c \leq 1$. We find that the resistivity works well as an order parmeter for the transition expected in analytic theory.

1. Introduction

The standard model has obtained excellent agreements with many experiments. Moreover the interactions at GUT's scale (m_{GUT}) seem to be unified and in the resion, m_{GUT} to m_{plancu} (Planck scale), we segard the theory as the unification including the quantum gravity. the string theory, which naturally contains gravity, the gauge theory, matter fields, SUSY and etc, is the most natural candidate of the quantum gravity. Nevertheless, according to the theory, we cannot help having too many perturbative vacua. Therefore, the non-perturbative treatment is needed in the string theory.

On the other hand recently there has been remarkable propress in the two-dimensional (2D) quantum gravity. The two analytic approaches have been estabished. The continuous approach is the Liouville field theory [1] [2]. The discretized approach is the matrix model [3]. We find the evidence for the equivalence of the two approaches. Moreover in general we consider that the 2D quantum gravity is equivalent to the non-critical string theory.

In the lattice formulation of the 2D quantum gravity, a discretized, numerical procedure has been constructed by using the dynamical triangulation (DT) method, in which calculations of the partition function are performed by replacing the path integral over the metric to a sum over possible triangulations. The studies of 2D surfaces are currently drawing much attention in variousfields of science such as physics, chemistry and biology. In physics, for example, statistical properties of triangulated surfaces [4] are under intensive investigation in the context of the 2D quantum gravity. The Monte Carlo simulations by the DT method reproduce surprisingly well fractal nature of the surface as predicted by the Liouville field theory [1][2] and the non-critical string field theory [5] [6].

In the DT method dual graphs of surfaces have direct correspondence to Feynman diagrams of ϕ^3 matrix model. The relation between the DT and the matrix model is evident. In case of the Liouville field theory it is expected that the the manifold in the continuous

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* 一般科

theory should be approximated by the DT method in the continum limit. In the present work, we review the recent topic of the fundamental relation between the DT method and critical strings. Defining the complex structure of a surface generated by the DT method and separating the conformal mode, we research the way to formulate the string analogue model (fishnet) non-perturbatively. We derive dual amplitudes from planar Feynman diagrams in the large order limit by the procedure of the electrical circuit analogy. The network to corresponding to a fine planar Feynman diagram can be regarded as a uniform homogeneous conducting sheet with a constant resistivity [7].

The organization of this paper is as follows. In the following section 2, we dedicate a brief review of the main results from the 2D lattice quantum gravity by the dynamical triangulation method. In section 3, we here discuss a method to define the complex structure and separate the conformal mode for a surface constructed by the 2D dynamical triangulation. We consider the way to formulate critical strings non-perturbatively.

2. Dynamical Triangulation Method

The 2D quantum gravity gives both the simplest model of Einstein gravity and a general framework for researching the universal properties of 2D surfaces. The DT method [4][8] is employed for the quantum gravity mainly with 2D simulations. In the DT approach we replace the path integral over the infinite dimensional space of metrics and some matter fields by a sum over all possible triangulations of space and matter configuration.

$$\int Dg^{\mu\nu}DX \Rightarrow \sum_{(tri),(m.c.)} \tag{1}$$

where (tri) and (m.c.) are triangulations and matter configurations, respectively. We can generate all configurations for the 2D or 3D case by the set of some triangle moves. The 4D version is, however, delicate and unclear in general. A triangle move can be defined in arbitary dimensions ($d=2, 3, 4$). In practice we often take a specific set of moves, which is called the (p, q) moves or the i move where i equals to $d+1-p$. Here i corresponds to the simplex dimension. The (p, q) moves generally conserve the topology. For instance, both the (1, 3) and (2, 2) moves conserve the topology, and are known to be ergodic, i.e. any two graphs of the same topology are connected by a sequence of these two moves. Let us now regard $N_i (i=0, 1, 2)$ as the total number of the i-simplices forming the surface. Then they are related by the following relationship,

$$N_2 - N_1 + N_0 = \chi, \ (Euler - Poincare) \tag{2}$$

$$3N_2 = 2N_1, \tag{3}$$

where the relation (2) is from the topological relationships and the relation (3) is specific to tiangulations. In our simulation we discretize a surface of area A by triangulation.

$$A = a^2 N_2, \tag{4}$$

where N_2 is the total number of 2-simplices and a^2 is the area of elementary triangle. In the DT method the Einstein-Hilbert action can be replaced as follows,

$$\int d^2x\sqrt{g}R=\sum_{hinge.i} 2\delta_i, \tag{5}$$

where δ_i is a deficit angle localized around a hinge i,

$$\delta_i=2\pi\left(\frac{1-q_i}{6}\right). \tag{6}$$

Here we denote q_i and R_i as the number of triangles sharing the a hinge i (the coordination number) ard the local curvature, respectively. Therefore the following relationship is shown,

$$\sum_{hinge.i} 2\delta_i=2\sum_i R_iA_i=\frac{2\pi}{3}\sum_i (6-q_i), \tag{7}$$

where A_i is a area attached to a hinge i,

$$A_i=\frac{1}{3}q_ia^2. \tag{8}$$

As a result the local curvature is obtained by

$$R_i=\frac{2\pi(6-q_i)}{a^2q_i}. \tag{9}$$

Let us now consider the case of the micro-canonical simulations. The two arbitary triangulated surfaces are connected by a (2, 2) move which means ergodic. Some sets of initial cofigurations are prepared where the topologies are fixed at a shere S^2, a torus T^2 and a *genus*=2 surface. We explein the procedure of the canonical DT of a 2D surface as follows,

- We prepare equilateral 3-simplices (tetrahedrons), and put them together by gluing triangles face- to-face forming a closed surface with the desired topology.
- For increasing the surface area, we glue more 3-simplices on the triangles of the closed surface, which corresponds to the barycentric subdvisions, or sometimes is called the (1, 3) move.
- To change the graphs, we pick up a link ramdomly and make the filipflop move or the (2, 2) move.

For the Monte Carlo simulation of the pure gravity, we take all graphs generated by the(2, 2) moves with equal weight as members of ensemble. the surface is expected to show factal behaviour because there are no dimensional parameters in this simulation of the pure gravity.

3. Complex Structure on DT surfaces

In this section we consider the new analogue model [7] for critical strings. One of the possible ways to formulate strings non-perturbatively is the string theory described by the local field. Now the fishnet model is introduced as an anlogue model. This model comes from attempts to derive dual amplitudes from planar Feynamn diagrans in the large ordar limit [9]. We make use of the electrical circuit analogy. The basis assumption of the method is that the network correspondsb to a fine planar Feynman diagram can be regarded as a uniform homogeneous conducting sheet with a constant resistivity. The basic idea is that the system looks like a string theory if fine planar Feynman diagrams dominate. In the continuum limit the fine network corresponding to the world-lines should be approximated to the surface corresponding to the world-sheet for strings.

First we define a fluctuating metric $g_{\mu\nu}$ on a 2D continuous surface. The metric can be decomposed into its complex structure.

$$g_{\mu\nu}=\hat{g}_{\mu\nu}(\tau,\ z)e^{\phi(z)}, \tag{10}$$

where τ is moduli and $\phi(z)$ is a conformal mode. In the DT method we consider that dynamical triangulated surfaces corresponds continuous surfaces in the large limit (the number of triangles). Practically these surfaces are known to be fractal [5]. We give a 2D conducting medium with conductivity tensor, $\sigma^{\mu\nu}$. We know that the Joule heat Q is generated by a potencial distribution V on the surface as follows,

$$Q=\int d^2\sigma^{\mu\nu}\partial_\mu V\partial_\nu V. \tag{11}$$

Now we require

$$\delta Q=0, \tag{12}$$

and the following equation is derived,

$$j^u=\sigma^{\mu\nu}\partial_\nu V, \tag{13}$$

where this formula shows the equation of continuity. Consequently by identifying $\partial_\nu V$ to be the electric field, the Ohm's law is obtained as follows,

$$j^u=\frac{1}{r}\sqrt{g}\,g^{\mu\nu}\partial_\nu V, \tag{14}$$

wher r is the resistivity constant. The above law in 2D is invariant under the local scale transformations,

$$g_{\mu\nu}\rightarrow g_{\mu\nu}e^{-\sigma}, \tag{15}$$

where we have an origin of complex structures defined on the DT surfaces.

Next we consider the complex structure on the 2D curved surfaces through measure-

ments of the resistivity. The above invariance property is also seen by using the resistance R of a small rectangular section of conducting sheet with length a and width b,

$$R=\frac{a}{b}r. \tag{16}$$

We find that the resistance R is invariant under the lacal scale transformation. We explain an algorithm to measure the resistivity r from the current and the voltage distributions. Four points are given on the surface with the complex coordinate, $z=x+iy$. The potential at point with the source of the current I placed at z_{in}, and a sink of the current at z_{out} is shown as follows,

$$V(z)=\frac{Ir}{2\pi}\,ln\left|\frac{z-z_{in}}{z-z_{out}}\right|+constant(\infty). \tag{17}$$

In order to cancel out this constant, we measure the potential drops between z_1 and z_2 as follows,

$$V(z_1)-V(z_2)=-\frac{Ir}{2\pi}\,ln\left|\frac{z_1-z_{in}}{z_1-z_{out}}\cdot\frac{z_2-z_{in}}{z_2-z_{out}}\right| \tag{18}$$

$$=-\frac{Ir}{2\pi}\,ln\,|\,[z_1,\ z_2\,;\ z_{in},\ z_{out}]\,|\,, \tag{19}$$

where $[z_1,\ z_2\,;\ z_{in},\ z_{out}]$ is known as the anharmonic ratio, and has $SL(2,\ C)$ invariance. Namely, under the projective transformation,

$$z\to\frac{az+b}{cz+d},\quad ad-bc=1, \tag{20}$$

the anharmonic ratio is invariant. By appropriately choosing four complex parameters, we can fix three points as follows,

$$z_{in}=0,\ z_{out}=\infty,\ z_2=1. \tag{21}$$

Therefore we get the potential drops,

$$V(z_1)-V(z_2)=-\frac{Ir}{2\pi}\,ln\,|\,z_1\,|\,, \tag{22}$$

$$V(z_2)-V(z_{in})=-\frac{Ir}{2\pi}\,ln\,|\,1-z_1\,|\,, \tag{23}$$

where the following conditions are satisfied, respectively,

$$[z_1,\ z_2\,;\ z_{in},\ z_{out}]=z_1, \tag{24}$$

$$[z_1,\ z_{in}\,;\ z_2,\ z_{out}]=1-z_1. \tag{25}$$

All the other possible combinations of the complex coordinate z give the linear combination of the above relationsphip. If the resistivity r is known, the complex coordinate z is determined. We regard $\{z,\ r\}$ as unknowns. We must add more other point, the fifth point.

We can measure numerically the reistivity r by getting the three extra equations. Namely if we have five equations for five unknowns, $\{z_i, z_j, r\}$, the resistivity is determined. If the sheet is made of a uniform homogeneous medium with the constant resistivity, the value of the resistivity r determined from the $V(z)$'s does not depend on the choice of the five electrodes.

We apply this algorithm to a random surface generated by the DT method. We regard the dual graphs of the surface consisting of N triangles as a trivalent network. We describe the numerical justification of the basic assertion as follows,

- We take a random trivalent network from the $c=0$ DT and assume each bond has the resistance, 1Ω, where c is the central charge.
- We take vertices randomly and calculate the $V(z)$'s.
- We determine the resistivity r from the $V(z)$'s.

The DT method generates the radom surfaces, where we fix the topology on S^2 and use a fixed number of triangles prohibiting the tadpole and selfenergy diagrams. We take a look the measurement for the pure gravity. As a result of the simulations, the distributions of the resistivity r for three different lattice sizes gives the distinct peaks at 2.6. The peaks get narrower as the size grows. We compare the value of the resistivity $r \approx 2.6$ to $\sqrt{3}$ of the flat network, where we have 6 triangles around each apex. When we get a large number of triangles, the peak grows infinitely as we expect in the continuum limit of a network of resister. We also find the tendency in the 1-Ising case ($c=1/2$) and the 1-scalar case ($c=1$). According to the result by the simulations [7], in the area out of the $c=1$ barrier, namely $c>1$, the surfaces are branched polymers and no continuum limit exists.

Recently the simplicial quantum gravity has been proposed as a regularization for the 4D quantum gravity [10]. The partition function is constructed by performing a weighted num over all triangulations of 4-sphere. The model is well-defined only if the number of such triangulated consisting of N simplices is exponentially bounded. Numerical simulations seem so far to favor such a bound. In addition, recent models for the discrete Euclidian quantum gravity incorporate a sum over simplicial triangulations. The algorithm for simulating such models leen described in the arbitrary dimension [11]. Moreover our project with the theory group of KEK [12] now proposes that the 4D generalized model [13] in the Liouville Field Theory (DDK) [2] should be simulated by the procedure of the improved DT method. On the other hand as for the simulations for the string theory, we understand that the Monte Carlo calculations are carried out for the 2D random surfaces coupled to matter fields such as 26 scalar fields and 51 Ising spins.

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References

[1] V. Knizhnik, A. Polyakov and A. Zamolodchikov, Mod. Phys. Lett. **A3** (1988) 819.
H.S. Egawa and Y. Fukuda, Res. Rept. of KIT. **B-16** (1992) 381.
[2] F. David, Mod. Phys. Lett. **A3** (1988) 651.
J. Distler and H. Kawai, Nucl. Phys. **B321** (1989) 506.
[3] E. Brezin and V. Kazakov, Phys. Lett. **B236** (1990) 144.
D. Gross and A. Migdal, Phys. Rev. Lett. **64** (1990) 127.
[4] J. Ambjørn, B. Durhuus and J. Frohlich, Nucl. Phys. **B257** (1985) 433.
F. David, Nucl. Phys. **B259** (1985) 45.
N. Tsuda, Doctor Thesis (1994) TIT.
[5] H. Kawai, N. Kawamoto, T. Mogami and Y. Watabiki, Phys. Lett. **B306** (1993) 19.
[6] N. Ishibashi and H. Kawai, Phys. Lett. **B314** (1993) 190.
N. Ishibashi and H. Kawai, Phys. Lett. **B322** (1994) 67.
H.S. Egawa and Y. Fukuda, Res. Rept. of KIT. **B-19** (1995) 189.
[7] H. Kawai, N. Tsuda and T. Yukawa, KEK-TH-**429** (1995) March.
[8] F. David, Lectures given at Les Houches (1992) "Simplicial Quantum Gravity and Random Lattices".
[9] B. Sakita and M. Virasoro, Phys. Rev. Lett. **24** (1970) 1146.
H. Nielsen and P. Olesen, Phys. Lett. B32 (1970) 203.
[10] J. Ambjørn and J. Jurkiewicz, NBI-HE-**94-29** May (1994).
[11] S. Catterall, CERN-TH. **7286/94** (1994).
[12] H.S.E, collaborations with N. Tsuda, T. Yukawa and H. Kawai (KEK).
[13] C. Schmidhuber, Nucl. Phys. **B390** (1993) 188.