# The primitive gap sequences at points on nonsingular curves of genus 9

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**Abstract**

Let $L$ be a *gap sequence*, i.e., a finite subset of the additive semigroup $\mathbf{N}$ of non-negative integers whose complement $\mathbf{N}\backslash L$ in $\mathbf{N}$ forms a subsemigroup of $\mathbf{N}$. Then the order of the set $L$ is called its *genus*. If twice the smallest positive integer of $\mathbf{N}\backslash L$ is larger than the largest integer of $L$, we say that $L$ is *primitive*. It is known that any gap sequence $L$ of genus $\leq 7$ (resp. any primitive gap sequence $L$ of genus 8) is *Weierstrass*[1], i.e., there exists a pointed curve $(C, P)$ such that $\mathbf{N}\backslash L$ is the set $L(P)$ of integers which are pole orders at $P$ of regular functions on $C\backslash\{P\}$ where a *curve* means a complete non-singular irreducible algebraic curve over an algebraically closed field of characteristic 0. Moreover, we showed that any non-primitive gap sequence of genus 8 except four sequences is Weierstrass[2]. In this paper we will show that any primitive gap sequence of genus 9 except only one sequence is Weierstrass.

Key Words: Gap sequence, Curve, Moduli

## §1. On primitive gap sequences of genus 9.

For a gap sequence $L = \{l_0 < l_1 < \cdots < l_{g-1}\}$ of genus $g$, $H(L)$ denotes the complement $\mathbf{N}\backslash L$ of $L$ in $\mathbf{N}$, which is a subsemigroup of $\mathbf{N}$. Let $M(L)$ be the minimal set of generators for $H(L)$. We set

$$\alpha(L) = (\alpha_0(L), \alpha_1(L), \cdots, \alpha_{g-1}(L)),$$

where $\alpha_i(L) = l_i - i - 1$ for any $i = 0, 1, \cdots, g-1$. Moreover, we set $w(L) = \sum_{i=0}^{g-1} \alpha_i(L)$,which is called the *weight* of $L$. Now we denote $\mathrm{Min}\{h \in H(L) | h > 0\}$ by $a(L)$. Then we have $2 \leq a(L) \leq g+1$. If $a(L) \leq 5$ or $a(L) \geq g$, then $L$ is Weierstrass[3),4),5),6)]. Hence we give the following table which shows the gap seqeunces $L$ of genus 9 with $a(L) = 6$ or 7 or 8, where $P$ (resp. $N$) means that $L$ is primitive (resp. non-primitive).

| | $L$ | $M(L)$ | $\alpha(L)$ | $w(L)$ | *Property* |
|---|---|---|---|---|---|
| (1) | $\{1,2,3,4,5,9,10,11,17\}$ | $\{6,7,8\}$ | $(0^5,3^3,8)$ | 17 | $N$ |
| (2) | $\{1,2,3,4,5,8,10,11,17\}$ | $\{6,7,9\}$ | $(0^5,2,3^2,8)$ | 16 | $N$ |
| (3) | $\{1,2,3,4,5,8,9,11,15\}$ | $\{6,7,10\}$ | $(0^5,2^2,3,6)$ | 13 | $N$ |
| (4) | $\{1,2,3,4,5,8,9,10,16\}$ | $\{6,7,11,15\}$ | $(0^5,2^3,7)$ | 13 | $N$ |
| (5) | $\{1,2,3,4,5,8,9,10,15\}$ | $\{6,7,11,16\}$ | $(0^5,2^3,6)$ | 12 | $N$ |
| (6) | $\{1,2,3,4,5,8,9,10,11\}$ | $\{6,7,15,16,17\}$ | $(0^5,2^4)$ | 8 | $P$ |
| (7) | $\{1,2,3,4,5,7,10,11,13\}$ | $\{6,8,9,19\}$ | $(0^5,1,3^2,4)$ | 11 | $N$ |
| (8) | $\{1,2,3,4,5,7,9,13,15\}$ | $\{6,8,10,11\}$ | $(0^5,1,2,5,6)$ | 14 | $N$ |
| (9) | $\{1,2,3,4,5,7,9,11,17\}$ | $\{6,8,10,13,15\}$ | $(0^5,1,2,3,8)$ | 14 | $N$ |
| (10) | $\{1,2,3,4,5,7,9,11,15\}$ | $\{6,8,10,13,17\}$ | $(0^5,1,2,3,6)$ | 12 | $N$ |
| (11) | $\{1,2,3,4,5,7,9,11,13\}$ | $\{6,8,10,15,17,19\}$ | $(0^5,1,2,3,4)$ | 10 | $N$ |

| | $L$ | $M(L)$ | $\alpha(L)$ | $w(L)$ | *Property* |
|---|---|---|---|---|---|
| (12) | $\{1,2,3,4,5,7,9,10,15\}$ | $\{6,8,11,13\}$ | $(0^5,1,2^2,6)$ | 11 | $N$ |
| (13) | $\{1,2,3,4,5,7,9,10,13\}$ | $\{6,8,11,15\}$ | $(0^5,1,2^2,4)$ | 9 | $N$ |
| (14) | $\{1,2,3,4,5,7,9,10,11\}$ | $\{6,8,13,15,17\}$ | $(0^5,1,2^3)$ | 7 | $P$ |
| (15) | $\{1,2,3,4,5,7,8,13,14\}$ | $\{6,9,10,11\}$ | $(0^5,1^2,5^2)$ | 12 | $N$ |
| (16) | $\{1,2,3,4,5,7,8,11,17\}$ | $\{6,9,10,13,14\}$ | $(0^5,1^2,3,8)$ | 13 | $N$ |
| (17) | $\{1,2,3,4,5,7,8,11,14\}$ | $\{6,9,10,13,17\}$ | $(0^5,1^2,3,5)$ | 10 | $N$ |
| (18) | $\{1,2,3,4,5,7,8,11,13\}$ | $\{6,9,10,14,17\}$ | $(0^5,1^2,3,4)$ | 9 | $N$ |
| (19) | $\{1,2,3,4,5,7,8,10,16\}$ | $\{6,9,11,13,14\}$ | $(0^5,1^2,2,7)$ | 11 | $N$ |
| (20) | $\{1,2,3,4,5,7,8,10,14\}$ | $\{6,9,11,13,16\}$ | $(0^5,1^2,2,5)$ | 9 | $N$ |
| (21) | $\{1,2,3,4,5,7,8,10,13\}$ | $\{6,9,11,14,16,19\}$ | $(0^5,1^2,2,4)$ | 8 | $N$ |
| (22) | $\{1,2,3,4,5,7,8,10,11\}$ | $\{6,9,13,16,17\}$ | $(0^5,1^2,2^2)$ | 6 | $P$ |
| (23) | $\{1,2,3,4,5,7,8,9,15\}$ | $\{6,10,11,13,14\}$ | $(0^5,1^3,6)$ | 9 | $N$ |
| (24) | $\{1,2,3,4,5,7,8,9,14\}$ | $\{6,10,11,13,15\}$ | $(0^5,1^3,5)$ | 8 | $N$ |
| (25) | $\{1,2,3,4,5,7,8,9,13\}$ | $\{6,10,11,14,15,19\}$ | $(0^5,1^3,4)$ | 7 | $N$ |
| (26) | $\{1,2,3,4,5,7,8,9,11\}$ | $\{6,10,13,14,15,17\}$ | $(0^5,1^3,2)$ | 5 | $P$ |
| (27) | $\{1,2,3,4,5,7,8,9,10\}$ | $\{6,11,13,14,15,16\}$ | $(0^5,1^4)$ | 4 | $P$ |
| (28) | $\{1,2,3,4,5,6,11,12,13\}$ | $\{7,8,9,10\}$ | $(0^6,4^3)$ | 12 | $P$ |
| (29) | $\{1,2,3,4,5,6,10,12,13\}$ | $\{7,8,9,11\}$ | $(0^6,3,4^2)$ | 11 | $P$ |
| (30) | $\{1,2,3,4,5,6,10,11,13\}$ | $\{7,8,9,12\}$ | $(0^6,3^2,4)$ | 10 | $P$ |
| (31) | $\{1,2,3,4,5,6,10,11,12\}$ | $\{7,8,9,13,19\}$ | $(0^6,3^3)$ | 9 | $P$ |
| (32) | $\{1,2,3,4,5,6,9,12,13\}$ | $\{7,8,10,11\}$ | $(0^6,2,4^2)$ | 10 | $P$ |
| (33) | $\{1,2,3,4,5,6,9,11,13\}$ | $\{7,8,10,12\}$ | $(0^6,2,3,4)$ | 9 | $P$ |
| (34) | $\{1,2,3,4,5,6,9,11,12\}$ | $\{7,8,10,13,19\}$ | $(0^6,2,3^2)$ | 8 | $P$ |
| (35) | $\{1,2,3,4,5,6,9,10,17\}$ | $\{7,8,11,12,13\}$ | $(0^6,2^2,8)$ | 12 | $N$ |
| (36) | $\{1,2,3,4,5,6,9,10,13\}$ | $\{7,8,11,12,17\}$ | $(0^6,2^2,4)$ | 8 | $P$ |
| (37) | $\{1,2,3,4,5,6,9,10,12\}$ | $\{7,8,11,13,17\}$ | $(0^6,2^2,3)$ | 7 | $P$ |
| (38) | $\{1,2,3,4,5,6,9,10,11\}$ | $\{7,8,12,13,17,18\}$ | $(0^6,2^3)$ | 6 | $P$ |
| (39) | $\{1,2,3,4,5,6,8,13,15\}$ | $\{7,9,10,11,12\}$ | $(0^6,1,5,6)$ | 12 | $N$ |
| (40) | $\{1,2,3,4,5,6,8,12,15\}$ | $\{7,9,10,11,13\}$ | $(0^6,1,4,6)$ | 11 | $N$ |
| (41) | $\{1,2,3,4,5,6,8,12,13\}$ | $\{7,9,10,11,15\}$ | $(0^6,1,4^2)$ | 9 | $P$ |
| (42) | $\{1,2,3,4,5,6,8,11,15\}$ | $\{7,9,10,12,13\}$ | $(0^6,1,3,6)$ | 10 | $N$ |
| (43) | $\{1,2,3,4,5,6,8,11,13\}$ | $\{7,9,10,12,15\}$ | $(0^6,1,3,4)$ | 8 | $P$ |
| (44) | $\{1,2,3,4,5,6,8,11,12\}$ | $\{7,9,10,13,15\}$ | $(0^6,1,3^2)$ | 7 | $P$ |
| (45) | $\{1,2,3,4,5,6,8,10,17\}$ | $\{7,9,11,12,13,15\}$ | $(0^6,1,2,8)$ | 11 | $N$ |
| (46) | $\{1,2,3,4,5,6,8,10,15\}$ | $\{7,9,11,12,13,17\}$ | $(0^6,1,2,6)$ | 9 | $N$ |
| (47) | $\{1,2,3,4,5,6,8,10,13\}$ | $\{7,9,11,12,15,17\}$ | $(0^6,1,2,4)$ | 7 | $P$ |
| (48) | $\{1,2,3,4,5,6,8,10,12\}$ | $\{7,9,11,13,15,17,19\}$ | $(0^6,1,2,3)$ | 6 | $P$ |
| (49) | $\{1,2,3,4,5,6,8,10,11\}$ | $\{7,9,12,13,15,17\}$ | $(0^6,1,2^2)$ | 5 | $P$ |
| (50) | $\{1,2,3,4,5,6,8,9,16\}$ | $\{7,10,11,12,13,15\}$ | $(0^6,1^2,7)$ | 9 | $N$ |
| (51) | $\{1,2,3,4,5,6,8,9,15\}$ | $\{7,10,11,12,13,16\}$ | $(0^6,1^2,6)$ | 8 | $N$ |
| (52) | $\{1,2,3,4,5,6,8,9,13\}$ | $\{7,10,11,12,15,16\}$ | $(0^6,1^2,4)$ | 6 | $P$ |
| (53) | $\{1,2,3,4,5,6,8,9,12\}$ | $\{7,10,11,13,15,16,19\}$ | $(0^6,1^2,3)$ | 5 | $P$ |
| (54) | $\{1,2,3,4,5,6,8,9,11\}$ | $\{7,10,12,13,15,16,18\}$ | $(0^6,1^2,2)$ | 4 | $P$ |
| (55) | $\{1,2,3,4,5,6,8,9,10\}$ | $\{7,11,12,13,15,16,17\}$ | $(0^6,1^3)$ | 3 | $P$ |
| (56) | $\{1,2,3,4,5,6,7,14,15\}$ | $\{8,9,10,11,12,13\}$ | $(0^7,6^2)$ | 12 | $P$ |
| (57) | $\{1,2,3,4,5,6,7,13,15\}$ | $\{8,9,10,11,12,14\}$ | $(0^7,5,6)$ | 11 | $P$ |
| (58) | $\{1,2,3,4,5,6,7,13,14\}$ | $\{8,9,10,11,12,15\}$ | $(0^7,5^2)$ | 10 | $P$ |

| | $L$ | $M(L)$ | $\alpha(L)$ | $w(L)$ | *Property* |
|---|---|---|---|---|---|
| (59) | $\{1,2,3,4,5,6,7,12,15\}$ | $\{8,9,10,11,13,14\}$ | $(0^7,4,6)$ | 10 | $P$ |
| (60) | $\{1,2,3,4,5,6,7,12,14\}$ | $\{8,9,10,11,13,15\}$ | $(0^7,4,5)$ | 9 | $P$ |
| (61) | $\{1,2,3,4,5,6,7,12,13\}$ | $\{8,9,10,11,14,15\}$ | $(0^7,4^2)$ | 8 | $P$ |
| (62) | $\{1,2,3,4,5,6,7,11,15\}$ | $\{8,9,10,12,13,14\}$ | $(0^7,3,6)$ | 9 | $P$ |
| (63) | $\{1,2,3,4,5,6,7,11,14\}$ | $\{8,9,10,12,13,15\}$ | $(0^7,3,5)$ | 8 | $P$ |
| (64) | $\{1,2,3,4,5,6,7,11,13\}$ | $\{8,9,10,12,14,15\}$ | $(0^7,3,4)$ | 7 | $P$ |
| (65) | $\{1,2,3,4,5,6,7,11,12\}$ | $\{8,9,10,13,14,15\}$ | $(0^7,3^2)$ | 6 | $P$ |
| (66) | $\{1,2,3,4,5,6,7,10,15\}$ | $\{8,9,11,12,13,14\}$ | $(0^7,2,6)$ | 8 | $P$ |
| (67) | $\{1,2,3,4,5,6,7,10,14\}$ | $\{8,9,11,12,13,15\}$ | $(0^7,2,5)$ | 7 | $P$ |
| (68) | $\{1,2,3,4,5,6,7,10,13\}$ | $\{8,9,11,12,14,15\}$ | $(0^7,2,4)$ | 6 | $P$ |
| (69) | $\{1,2,3,4,5,6,7,10,12\}$ | $\{8,9,11,13,14,15\}$ | $(0^7,2,3)$ | 5 | $P$ |
| (70) | $\{1,2,3,4,5,6,7,10,11\}$ | $\{8,9,12,13,14,15,19\}$ | $(0^7,2^2)$ | 4 | $P$ |
| (71) | $\{1,2,3,4,5,6,7,9,17\}$ | $\{8,10,11,12,13,14,15\}$ | $(0^7,1,8)$ | 9 | $N$ |
| (72) | $\{1,2,3,4,5,6,7,9,15\}$ | $\{8,10,11,12,13,14,17\}$ | $(0^7,1,6)$ | 7 | $P$ |
| (73) | $\{1,2,3,4,5,6,7,9,14\}$ | $\{8,10,11,12,13,15,17\}$ | $(0^7,1,5)$ | 6 | $P$ |
| (74) | $\{1,2,3,4,5,6,7,9,13\}$ | $\{8,10,11,12,14,15,17\}$ | $(0^7,1,4)$ | 5 | $P$ |
| (75) | $\{1,2,3,4,5,6,7,9,12\}$ | $\{8,10,11,13,14,15,17\}$ | $(0^7,1,3)$ | 4 | $P$ |
| (76) | $\{1,2,3,4,5,6,7,9,11\}$ | $\{8,10,12,13,14,15,17,19\}$ | $(0^7,1,2)$ | 3 | $P$ |
| (77) | $\{1,2,3,4,5,6,7,9,10\}$ | $\{8,11,12,13,14,15,17,18\}$ | $(0^7,1^2)$ | 2 | $P$ |

We know that any primitive gap sequence of genus $g$ and weight $\leq g-1$ is Weierstrass[7),8)]. Hence the primitive gap sequences of the above table except the sequences (28),(29),(30),(31),(32),(33), (41),(56),(57),(58),(59),(60) and (62) are Weierstrass. Moreover, any primitive gap sequence $L$ of genus $g$ and weight $g$ with $\alpha(L) = (0^{g-2}, m, n)$ is Weierstrass[1)]. Hence the gap sequences (60) and (62) are Weierstrass. Now S.J. Kim[9)] showed that for any gap sequence $L$ with $\alpha(L) = (0^{g-r}, m^r)$ there exists a pointed trigonal curve $(C, P)$ such that $L(P) = L$. Therefore the gap sequences (28),(31),(56) and (58) are Weierstrass.

## §2. 1-neat gap sequences.

In this section we are devoted to the following gap sequences :

**Definitin 2.1** Let $L$ be a gap sequence with $M(L) = \{a_1, a_2, a_3, a_4\}$. We set

$$\alpha_i = \mathrm{Min}\{\alpha \in \mathbf{N}\backslash\{0\} \mid \alpha a_i \in < a_1, \cdots, a_{i-1}, a_{i+1}, \cdots, a_4 >\}$$

for all $i = 1,2,3,4$. Then the gap sequence $L$ is said to be 1-*neat* if renumbering $a_1, a_2, a_3, a_4$ there exist non-negative integers $\alpha_{ij}$ $(i \neq j, 1 \leq i \leq 4, 1 \leq j \leq 4)$ which satisfy the following :

$$\alpha_i a_i = \sum_{j=1, j\neq i}^{4} \alpha_{ij} a_j, 0 \leq \alpha_{ij} < \alpha_j, \quad (1 \leq i \leq 4), \quad \sum_{i=1, i\neq j}^{4} \alpha_{ij} = \alpha_j \quad (1 \leq j \leq 4)$$

and

$$\det \begin{pmatrix} \alpha_1 & -\alpha_{12} & -\alpha_{13} \\ -\alpha_{21} & \alpha_2 & -\alpha_{23} \\ -\alpha_{31} & -\alpha_{32} & \alpha_3 \end{pmatrix} = a_4.$$

Then the following holds[10)].

**Remark 2.2** Any 1-neat gap sequence is Weierstrass.

In the remainder of this section we shall show that the gap sequences (29),(30) and (33) in the table of §1 are 1-neat.

**Proposition 2.3** *The gap sequence* (29) *in the table of* §*1 is 1-neat, hence it is Weierstrass.*

*Proof.* Let $L = \{1,2,3,4,5,6,10,12,13\}$. Then $M(L) = \{7,8,9,11\}$. We set $a_1 = 7$, $a_2 = 8$, $a_3 = 9$ and $a_4 = 11$. Then we have the following relations :

$$4a_1 = a_2 + a_3 + a_4, \quad 2a_2 = a_1 + a_3, \quad 2a_3 = a_1 + a_4 \quad \text{and} \quad 2a_4 = 2a_1 + a_2,$$

which imply that $\alpha_1 = 4$, $\alpha_2 = 2$, $\alpha_3 = 2$ and $\alpha_4 = 2$. Since we have

$$\det \begin{pmatrix} 4 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & 0 & 2 \end{pmatrix} = 11 = a_4,$$

the gap sequence $L$ is 1-neat. *Q.E.D.*

**Proposition 2.4** *The gap sequence* (30) *in the table of* §*1 is 1-neat, hence it is Weierstrass.*

*Proof.* Let $L = \{1,2,3,4,5,6,10,11,13\}$. Then $M(L) = \{7,8,9,12\}$. We set $a_1 = 7$, $a_2 = 8$, $a_3 = 9$ and $a_4 = 12$. Then we have the following relations :

$$3a_1 = a_3 + a_4, \quad 2a_2 = a_1 + a_3, \quad 3a_3 = a_1 + a_2 + a_4 \quad \text{and} \quad 2a_4 = a_1 + a_2 + a_3,$$

which imply that $\alpha_1 = 3$, $\alpha_2 = 2$, $\alpha_3 = 3$ and $\alpha_4 = 2$. Since we have

$$\det \begin{pmatrix} 3 & 0 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 3 \end{pmatrix} = 12 = a_4,$$

the gap sequence $L$ is 1-neat. *Q.E.D.*

**Proposition 2.5** *The gap sequence* (33) *in the table of* §*1 is 1-neat, hence it is Weierstrass.*

*Proof.* Let $L = \{1,2,3,4,5,6,9,11,13\}$. Then $M(L) = \{7,8,10,12\}$. We set $a_1 = 7$, $a_2 = 8$, $a_3 = 10$ and $a_4 = 12$. Then we have the following relations :

$$4a_1 = 2a_2 + a_4, \quad 3a_2 = 2a_1 + a_3, \quad 2a_3 = a_2 + a_4 \quad \text{and} \quad 2a_4 = 2a_1 + a_3,$$

which imply that $\alpha_1 = 4$, $\alpha_2 = 3$, $\alpha_3 = 2$ and $\alpha_4 = 2$. Since we have

$$\det \begin{pmatrix} 4 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 2 \end{pmatrix} = 12 = a_4,$$

the gap sequence $L$ is 1-neat. *Q.E.D.*

## §3. Dimensionally proper gap sequences.

In this section we shall treat the following gap sequences :

**Definitin 3.1** Let $L$ be a gap sequence of genus $g$. Then we define a locally closed subset of $\mathcal{M}_{g,1}$ by

$$\mathcal{C}_L = \{(C,P) \in \mathcal{M}_{g,1} \mid L(P) = L\},$$

where $\mathcal{M}_{g,1}$ denotes the moduli space of pointed curves of genus $g$. Then the weight $w(L)$ of $L$ gives an upper bound for the codimension of any irreducible component of $\mathcal{C}_L$ in $\mathcal{M}_{g,1}$. Then $L$ is *dimensionally proper* if there exists an irreducible component of $\mathcal{C}_L$ of codimension $w(L)$, i.e., dimension $3g-2-w(L)$.

Using the theory of limit linear series Eisenbud and Harris[7)] showed the following which is useful for investigating whether a primitive gap sequence is dimensionally proper.

**Remark 3.2** Let $L$ be a dimensionally proper gap sequence of genus $g-1$ with $\alpha(L) = (\alpha_0, \alpha_1, \dots, \alpha_{g-2})$. Then the gap sequence $M$ with $\alpha(M) = (\beta_0, \beta_1, \dots, \beta_{g-1})$ is dimensionally proper if it satisfies one of the following :

1) $\beta_0 = 0,\ \beta_i = \alpha_{i-1}\ (i = 1, \dots, g-1)$,
2) for some $0 < j \leq g-1$, $\beta_0 = 0,\ \beta_j = \alpha_{j-1}+1,\ \beta_i = \alpha_{i-1}\ (i = 1, \dots, g-1, i \neq j)$.

**Proposition 3.3** *The gap sequences* (41), (57) *and* (59) *in the table of* §*1 are dimensionally proper, hence they are Weierstrass.*

*Proof.* Let $L_1$ be the gap sequence (41), i.e., $\alpha(L_1) = (0^6, 1, 4^2)$. Since the gap sequence $M_1$ of genus 8 with $\alpha(M_1) = (0^6, 4^2)$ is dimensionally proper[11)], by Remark 3.2 so is $L_1$. Let $L_2$ be the gap sequence (57), i.e., $\alpha(L_2) = (0^7, 5, 6)$. Since the gap sequence $M_2$ of genus 8 with $\alpha(M_2) = (0^6, 5^2)$ is dimensionally proper[12)], by Remark 3.2 so is $L_2$. Let $L_3$ be the gap sequence (59), i.e., $\alpha(L_3) = (0^7, 4, 6)$. Since the gap sequence $N_3$ of genus 7 with $\alpha(N_3) = (0^5, 4^2)$ is dimensionally proper[13)], by Remark 3.2 so is the gap sequence $M_3$ with $\alpha(M_3) = (0^6, 4, 5)$. Hence using Remark 3.2 again the gap sequence $L_3$ is dimensionally proper. *Q.E.D.*

By §1 and Propositions 2.3, 2.4, 2.5, 3.3 we get the follwoing which is the main theorem in this paper.

**Theorem 3.4** *Any primitive gap sequence of genus* 9 *except only one gap sequence* $\{1, 2, 3, 4, 5, 6, 9, 12, 13\}$ *is Weierstrass.*

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